SUBJECT — PHYSICS

Q.1/A (d) Photoelectric effect

Q.2/A (a) Speed of light

Q.3/A (a) Interference

Q.4/A Bright

Q.5/A Longitudinal wave

Q.6/A Infinite

Q.1/B According to the variation of mass with velocity, the relativistic mass m of the particle is given by

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{(c/\sqrt{2})^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{1}{2}}} = \sqrt{2}\, m_0 = 1.414\, m_0$$

The momentum p of the particle is given by

$$p = mv = m_0\sqrt{2} \times \frac{c}{\sqrt{2}} = m_0 c$$

The total energy E of the particle is given by

$$E = mc^2 = (1.414\, m_0)c^2 = 1.414\, m_0 c^2$$

The kinetic energy K of the particle is given by

$$K = E - mc^2 = 1.414\, m_0 c^2 - m_0 c^2 = 0.414\, m_0 c^2$$

Q.2/B By the law of addition of velocities, their relative speed u will be given by

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{0.7c + 0.7c}{1 + \left(\frac{0.7c \times 0.7c}{c^2}\right)^2} = \frac{1.4c}{1.49}$$

$$= 0.9396\, c = 2.819 \times 10^8\ m/s$$

Q.3/B The energy of a particle is given by

$$E = mc^2 \quad \text{or} \quad m = \frac{E}{c^2}$$

Here, $E = 2\ \text{MeV} = 2 \times 10^6 \times 1.6 \times 10^{-19}\ J$

and $c = 3 \times 10^8\ m/s$

$$\therefore\ m = \frac{2 \times 10^6 \times 1.6 \times 10^{-19}}{(3 \times 10^8)^2} = 3.55 \times 10^{-30}\ Kg.$$

According to variation of mass with velocity

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{or} \quad 1 - \frac{v^2}{c^2} = \left(\frac{m_0}{m}\right)^2$$

$$\text{or } v = c\sqrt{1 - \left(\frac{m_0}{m}\right)^2} = 3 \times 10^8 \times \sqrt{1 - \left(\frac{9.11 \times 10^{-31}}{3.55 \times 10^{30}}\right)^2}$$

$$= 3 \times 10^8 \times 0.967 = 2.90 \times 10^8\ m/s$$

Q.4/B According to time dilation

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Here, $t = 2$ day, $t_0 = 1$ day.

Hence $\dfrac{t}{t_0} = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$\text{or, } \frac{2}{1} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{or} \quad \sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2}$$

$$\Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4} \quad \text{or} \quad \frac{v^2}{c^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\therefore\ v = \frac{\sqrt{3}}{2}c = \frac{\sqrt{3}}{2} \times 3 \times 10^8\ m/s$$

$$= 2.598 \times 10^8\ m/s$$

Q.5/B The grating equation is

$$(a+b)\sin\theta = n\lambda$$

$$\text{or } \lambda = \frac{(a+b)\sin\theta}{n}$$

$\lambda = \lambda_{max}$ if $\theta = 90^\circ$ or $\sin\theta = 1$

Hence, $$\lambda_{max} = \frac{a+b}{n}$$

Here, grating element $(a+b) = \frac{1}{5000}$ cm

and order $n = 4$

$$\therefore \lambda_{max} = \frac{1}{5000} \times \frac{1}{4} \text{ cm}$$

$$= \frac{1}{20000} \text{ cm}$$

$$= 5 \times 10^{-5} \text{ cm}$$

$\therefore$ Required longest wavelength $= 5 \times 10^{-5}$ cm

Q.6/B The thickness of a quarter wave plate for positive crystal like quartz is

$$t = \frac{\lambda}{4(\mu_E - \mu_0)}$$

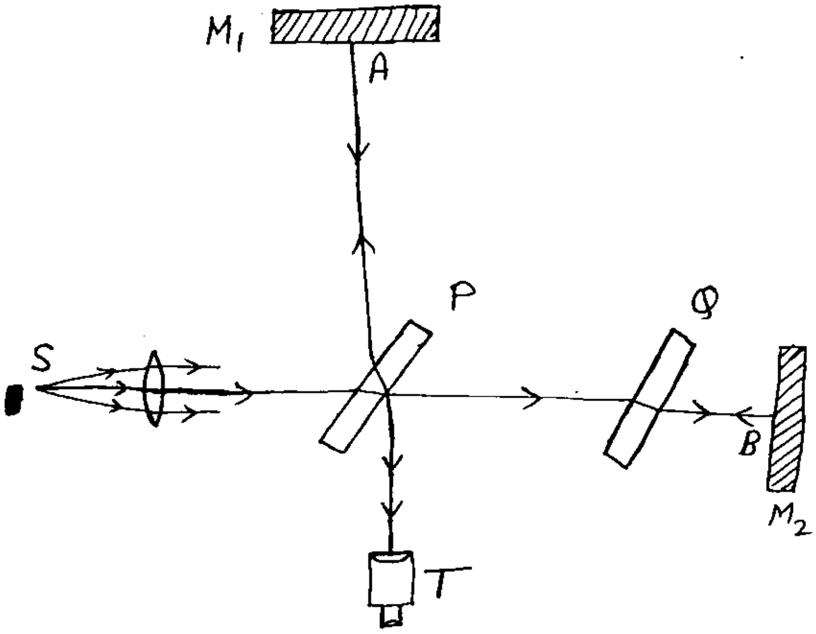
Here, $\mu_0 = 1.54425$, $\mu_E = 1.55336$, $\lambda = 5893\,\text{Å} = 5.893 \times 10^{-5}$ cm

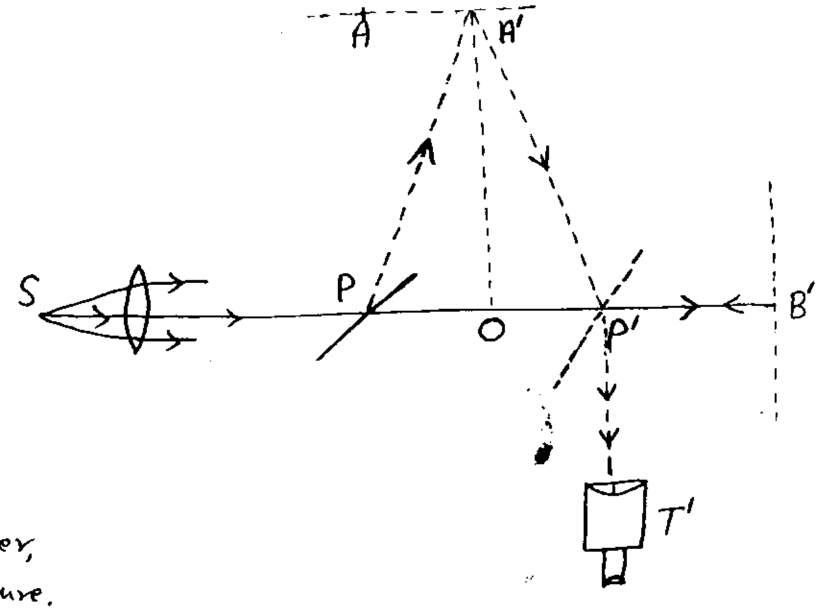
$$\therefore t = \frac{5.893 \times 10^{-5}}{4(1.55336 - 1.54425)} = \frac{5.893 \times 10^{-5}}{4 \times 911 \times 10^{-5}}$$

$$= 1.62 \times 10^{-3} \text{ cm}$$

Q.1/C The main objective of conducting the Michelson-Morley experiment was to confirm the existence of a stationary ether (frame). This experiment was performed for measuring the absolute velocity of the earth with respect to stationary ether.

- The essential features of the apparatus used, universally known as the Michelson interferometer, are shown in the figure.





It consists of a monochromatic light source S, half-silvered plate P, inclined at 45° to the beam of light and two mirrors M_1 and M_2, highly silvered on front surfaces to avoid multiple internal reflections. The compensating plate Q is used such that P and Q are of equal thickness and of the same material, mounted parallel to each other. The plate Q does not reflect light at all. The function of the plate Q is only to equalise the optical paths traversed by both the beams if mirrors M_1 and M_2 are at equal distances from P. Let ~~be~~ this equal distance be l.

If the apparatus is at rest in ether then the two rays (reflected and transmitted) would take equal time to return to P. But the whole apparatus is moving with earth through the ether with a velocity, say, v. Consider the direction of motion of earth in the direction of the initial beam of light. Due to motion of earth, the optical paths traversed by both the rays are not the same. Therefore, the reflections at mirror

M_1 and M_2 do not take place at A and B but at A' and B' respectively. The new position of plate P is at P' and plate Q is not shown in the second figure for convenience.

Total path traversed by reflected beam $= PA'P' = PA' + A'P' = 2PA'$

Also $(PA')^2 = (PO)^2 + (OA')^2$

If t = time taken by the beam to reach from P to M_1, then

$PA' = ct$ and $AA' = vt$ where c = velocity of light.

Thus, $c^2t^2 = v^2t^2 + l^2$ or $t = \dfrac{l}{\sqrt{c^2-v^2}}$

If the time t_1 is taken by the reflected ray to travel total path,

$$t_1 = 2t = \frac{2l}{\sqrt{c^2-v^2}} = \frac{2l}{c}\left(1-\frac{v^2}{c^2}\right)^{-1/2} = \frac{2l}{c}\left(1+\frac{v^2}{2c^2}\right)$$ using Binomial theorem

Total time taken by the transmitted beam to travel the total path, i.e. from P to B' and then B' to P' $= t_2$ (let).

$$\therefore t_2 = \frac{l}{c-v} + \frac{l}{c+v} = \frac{2lc}{c^2-v^2}$$

$$= \frac{2l}{c^2}\left(1-\frac{v^2}{c^2}\right)^{-1} = \frac{2l}{c}\left(1+\frac{v^2}{c^2}\right)$$ using Binomial theorem

The time difference, $\Delta t = t_2 - t_1 = \dfrac{2l}{c}\cdot\dfrac{v^2}{2c^2} = \dfrac{lv^2}{c^3}$

The path difference between the two rays

$$\Delta = c\Delta t = c\cdot\frac{lv^2}{c^3} = \frac{lv^2}{c^2}$$

Path difference in terms of number of fringes

$$= \frac{1}{\lambda}\cdot\frac{lv^2}{c^2} = \frac{lv^2}{c^2\lambda}$$ where λ = wavelength of light used.

In the actual experiment the whole apparatus was floating on mercury. The whole apparatus was turned through 90°. In this case the path difference of the same amount in opposite direction will be introduced. Hence a shift of $\dfrac{2l^2v^2}{c^2\lambda}$ was expected.

The expected fringe shift $\Delta n = \dfrac{2lv^2}{c^2}\cdot\dfrac{1}{\lambda}$

$$= \frac{2\times 11\times(3\times10^4)^2}{(3\times10^8)^2}\cdot\frac{1}{5.5\times10^{-7}} = 0.4$$

A shift of this magnitude can be easily measured by the interferometer.

Michelson and Morley were extremely surprised to see that no shift in the fringe was observed when the interferometer was rotated through 90°.

Q.2/C Let us consider a particle of mass m acted upon by a force F in the same direction as its velocity v. If the force F displaces the particle through a small distance ds, then work done, dW is stored by the particle as its kinetic energy dK. Therefore,

$$dK = dW = F.ds \quad \text{—— (1)}$$

According to Newton's law of motion, the force is defined as the rate of change of momentum p, that is,

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) \quad \text{—— (2)}$$

According to theory of relativity, mass of the particle varies with velocity. Hence both m and v are variable.

Therefore, $$F = m\frac{dv}{dt} + v\frac{dm}{dt} \quad \text{—— (3)}$$

Substituting in equation (1) we get

$$dK = m\frac{dv}{dt}ds + v\frac{dm}{dt}ds = m\frac{ds}{dt}dv + v\frac{ds}{dt}dm$$

or, $$dK = mv\,dv + v^2 dm \quad \left(\because \frac{ds}{dt} = v\right) \quad \text{—— (4)}$$

According to special theory of relativity, the mass m of a particle moving with velocity v varies in accordance with the relation

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = m_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} \quad \text{—— (5)}$$

where m_0 = rest mass of the particle.

Differentiating equation (5), we get

$$dm = m_0\left(-\frac{1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(-\frac{2v\,dv}{c^2}\right) = \frac{m_0 v\,dv}{c^2\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

But from equation (5), $m_0 = m\left(1-\frac{v^2}{c^2}\right)^{1/2}$

$$\therefore\ dm = \frac{m\left(1-\frac{v^2}{c^2}\right)^{1/2} v\,dv}{c^2\left(1-\frac{v^2}{c^2}\right)^{3/2}} = \frac{mv\,dv}{(c^2-v^2)}.$$

or, $mv\,dv = (c^2-v^2)\,dm$ —— (6)

Substituting this value of $mv\,dv$ in equation (4), we get

$$dK = (c^2-v^2)\,dm + v^2\,dm = c^2\,dm$$

If the change in kinetic energy of the particle be K, when its mass changes from rest mass m_o to effective mass m, then

$$K = \int dK = \int_{m_o}^{m} c^2\,dm = c^2(m-m_o)$$

- The total energy, E = Kinetic Energy K + rest mass energy

or, $E = K + m_oc^2 = (m-m_o)c^2 + m_oc^2$

or $\boxed{E = mc^2}$

This is the well known Einstein's mass-energy relation which states a universal equivalence between mass and energy.

Now, $E = mc^2 = \dfrac{m_oc^2}{\sqrt{1-\frac{v^2}{c^2}}}$

But momentum $p = mv$ i.e. $v = \frac{p}{m}$, hence

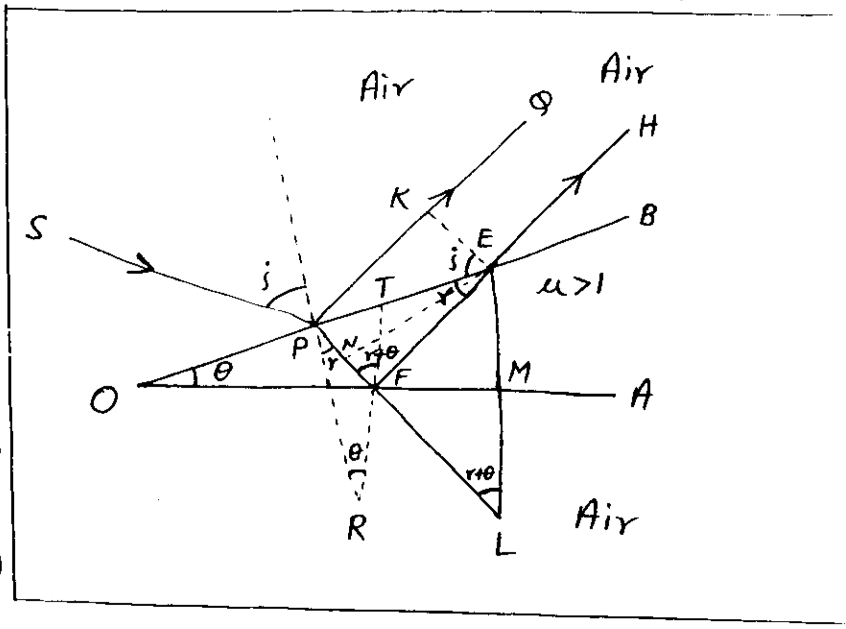
$$E = \frac{m_oc^2}{\sqrt{1-\frac{p^2}{m^2c^2}}} = \frac{m_oc^2}{\sqrt{1-\frac{p^2c^2}{(mc^2)^2}}}$$

or $E = \dfrac{m_oc^2}{\sqrt{1-\frac{p^2c^2}{E^2}}}$ $(\because E = mc^2)$

Squaring, $E^2 = \dfrac{m_o^2c^4}{1-\frac{p^2c^2}{E^2}}$ or $E^2\left(1-\frac{p^2c^2}{E^2}\right) = m_o^2c^4$

or, $\boxed{E^2 - p^2c^2 = m_o^2c^4}$ Proved.

Q.4/c Consider a film in the shape of a thin wedge whose sides are slightly inclined to each other at an angle θ and enclosing a film of transparent material of refractive index μ. The thickness of the film increases gradually from O to A as shown in the figure. When the film is illuminated by a beam of monochromatic light, then the interference between the two rays, one reflected (PQ) from the upper surface of the film and the other (EH) obtained by internal reflection at the back surface and consequent transmission at the first surface (AB), takes place and a system of equidistance alternate bright and dark fringes are observed. As the interfering rays PQ and EH are derived from the same incident ray SP, they are coherent. The optical path difference between these two rays is given by



$$\Delta = (PF + FE) \text{ in film} - PK \text{ in air}$$

$$= \mu (PF + FE) - PK = \mu (PN + NF + FE) - PK \quad \text{——} \ (1)$$

Now, $\mu = \dfrac{\sin i}{\sin r} = \dfrac{PK/PE}{PN/PE} = \dfrac{PK}{PN}$

$$\therefore \ PK = \mu PN \quad \text{——} \ (2)$$

Substituting in (1) we get

$$\Delta = \mu (PN + NF + FE) - \mu PN$$

or $$\Delta = \mu (NF + FE) \quad \text{——} \ (3)$$

In congruent triangles EFM and LFM we have

$$EM = ML = t \ \text{ and } \ FE = FL \quad \text{——} \ (4)$$

Hence from (3) we have

$$\Delta = \mu (NF + FL) = \mu NL \quad \text{——} \ (5)$$

But in triangle NEL, $NL = EL \cos(r+\theta)$. But $EL = 2EM = 2t$

$$\therefore \ NL = 2t \cos(r+\theta) \quad \text{——} \ (6)$$

Therefore from (5) we have

$$\text{path difference } \Delta = 2\mu t \cos(r+\theta)$$

As the ray PQ is the reflected ray from a denser medium, therefore, there occurs an additional path difference of $\lambda/2$ or phase change of π.

Thus effective path difference $= \Delta + \frac{\lambda}{2} = 2\mu t \cos(r+\theta) + \frac{\lambda}{2}$

For constructive interference or maxima,

$$2\mu t \cos(r+\theta) + \frac{\lambda}{2} = 2n \cdot \frac{\lambda}{2}$$

$$\text{or} \quad 2\mu t \cos(r+\theta) = (2n-1)\frac{\lambda}{2} \quad \text{where, } n = 1, 2, 3, \ldots \quad \text{---} (7)$$

For destructive interference or minima,

$$2\mu t \cos(r+\theta) + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

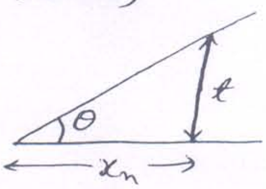
$$\text{or} \quad 2\mu t \cos(r+\theta) = n\lambda \quad \text{where } n = 0, 1, 2, 3, \ldots \quad \text{---} (8)$$

Fringe Width The seperation between the two successive bright fringes or dark fringes or fringe width may be obtained as follows:

For nth bright fringe, $2\mu t \cos(r+\theta) = (2n-1)\frac{\lambda}{2}$ --- (9)

Let x_n ~~bet~~ be the distance of nth bright fringe from the edge of the film, then we have (from figure)

$$\tan\theta = \frac{t}{x_n} \quad \text{or} \quad t = x_n \tan\theta$$



Hence from (9) we get —

$$2\mu x_n \tan\theta \cos(r+\theta) = (2n-1)\frac{\lambda}{2} \quad \text{---} (10)$$

Similarly if x_{n+1} is the distance of (n+1)th bright fringe,

$$2\mu x_{n+1} \tan\theta \cos(r+\theta) = [2(n+1)-1]\frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \quad \text{---} (11)$$

Substracting (10) from (11) we get

$$2\mu(x_{n+1} - x_n)\tan\theta \cos(r+\theta) = \lambda$$

Hence fringe width, $\omega = x_{n+1} - x_n = \dfrac{\lambda}{2\mu \tan\theta \cos(r+\theta)}$