Solution Physics Paper-1

Q1/A (c) Moving with uniform velocity or at rest

Q2/A (b) Transverse waves only

Q3/A (c) π

Q4/A Velocity

Q5/A plane wave front.

Q6/A For dark rings $D_n \propto \sqrt{n}$
For bright rings $D_n \propto \sqrt{2n-1}$ } where n is integer

Q1/B.

$m = 11 m_0$ $\quad$ m_0 = rest mass $\quad$ m = moving mass

$K = (m - m_0)c^2 = (11 m_0 - m_0)c^2$

$= 10 m_0 c^2$

$m_0 = 9.1 \times 10^{-31}$ kg $\quad c = 3 \times 10^8 \, ms^{-1}$

$\Rightarrow K = 10 \times 9.1 \times 10^{-31} \times 9 \times 10^{16}$ joules

$= 819 \times 10^{-15}$ J

$\boxed{K = 8.19 \times 10^{-13} \text{ J}}$ —— (1)

$p = mv = 11 m_0 v$ —— (2)

Since $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}} \Rightarrow 11 \cancel{m_0} = \dfrac{\cancel{m_0}}{\sqrt{1 - \frac{v^2}{c^2}}}$

$\Rightarrow 1 - \dfrac{v^2}{c^2} = \dfrac{1}{121} \Rightarrow \dfrac{v^2}{c^2} = 1 - \dfrac{1}{121} = \dfrac{120}{121}$

$v = \dfrac{c \cdot \sqrt{120}}{11}$

$\Rightarrow p = \cancel{11} m_0 \dfrac{\sqrt{120}}{\cancel{11}} c = 9.1 \times 10^{-31} \times 10.954 \times 3 \times 10^8$

$= 99.69 \times 10^{-31} c \text{ kg } ms^{-1}$

$= 9.969 \times 10^{-30} \cdot c \text{ kg } ms^{-1}$

$= 9.969 \times 3 \times 10^8 \times 10^{-30}$

$= 29.907 \times 10^{-22}$

$= 2.99 \times 10^{-21} \text{ kg } ms^{-1}$

Q2/B

$K = 2 m_0 c^2$ (Given)

$\Rightarrow (m - m_0)c^2 = 2 m_0 c^2$

$mc^2 = 3 m_0 c^2 \Rightarrow m = 3 m_0 \Rightarrow \dfrac{m_0}{\sqrt{1 - v^2/c^2}} = 3 m_0$

$\Rightarrow 1 - \dfrac{v^2}{c^2} = \dfrac{1}{9} \Rightarrow v^2/c^2 = \dfrac{8}{9} \Rightarrow v = \dfrac{2\sqrt{2}}{3} \cdot c$

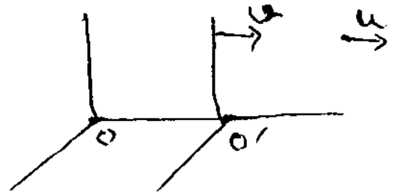
$v = \dfrac{2 \times 1.414 \times 3 \times 10^8}{3} = 2.828 \times 10^8 \, ms^{-1}$

Q3/B

$u = ?$

$v = 0.9c$

$u' = 0.5c$



$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{0.5c + 0.9c}{1 + \frac{0.5c \times 0.9c}{c^2}}$$

$$= \frac{1.4c}{1.45} = 0.9655c$$

$\underline{u = 0.9655c}$

Q4/B

$l = 0.99 l_0 \qquad v = ?$

$l = l_0 \sqrt{1 - v^2/c^2}$

$0.99 l_0 = l_0 \sqrt{1 - v^2/c^2}$

$(0.99)^2 = 1 - v^2/c^2 \Rightarrow 0.9801 = 1 - \frac{v^2}{c^2}$

$1 - 0.9801 = \frac{v^2}{c^2} \Rightarrow v = c \times .0199$

$\Rightarrow v = 3 \times 10^8 \times .0199 = 597 \times 10^4$

$\underline{v = 5.97 \times 10^6 \ ms^{-1}}$

Q5/B

$n_{max} = ? \qquad \lambda = 6000 \text{ Å} = 6 \times 10^{-7} m$

$(a + b) \sin\theta = n\lambda$

For $n = n_{max}$ $\sin\theta = 1$

$a + b = \frac{2.54}{16000}$ cm $\qquad$ 1 inch = 2.54 cm

$$= \frac{2.54 \times 10^{-2}}{16000} m$$

$\Rightarrow \frac{2.54 \times 10^{-5}}{16} \times 1 = n_{max} \cdot 6 \times 10^{-7}$

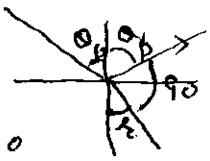
$\Rightarrow n_{max} = \frac{254 \times 10^{-7}}{16 \times 6 \times 10^{-7}} = \frac{254}{96}$

$= 2.645$

$\therefore$ Highest order spectrum that can be seen = $\underline{2}$

Q6/B

$\theta_p = ? \quad r = ? \quad \mu = 1.5$



$\mu = \tan\theta_p \Rightarrow \tan\theta_p = 1.5$

$\theta_p = \tan^{-1} 1.5 \Rightarrow \underline{\theta_p = 56.29^\circ}$

$\theta_p + r = 90 \Rightarrow r = 90 - \theta_p = 90 - 56.29$

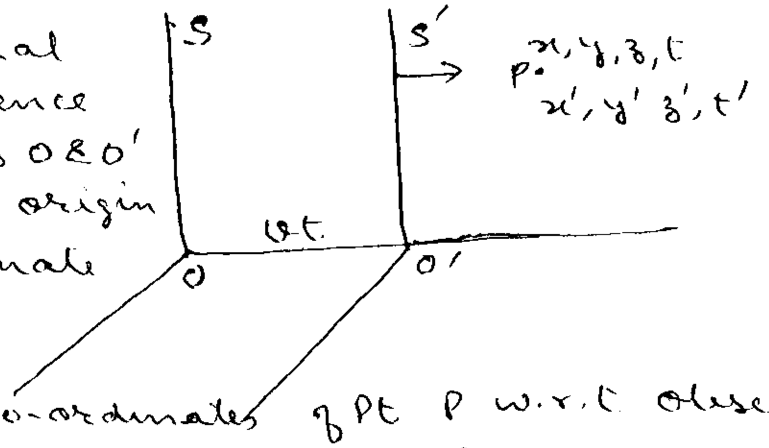
$r = \underline{23.71^\circ}$

Q1 c.

Lorentz transformations are the set of equations which are used to convert (transform) the set of co-ordinates (x', y', z', t') in a moving system to the set of co-ordinates (x, y, z, t) in a stationary system.

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad y' = y \quad z' = z \ \& \ t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - v^2/c^2}}$$

S & S' are inertial frames of reference with observers O & O' respectively at origin of their co-ordinate system.



(x, y, z, t) are co-ordinates of Pt P w.r.t observer O

x', y', z', t' are " " " P " " O'.

S' moves with uniform velocity v w.r.t S in +ve x-direction

Equations of spherical wave front from a point source w.r.t S & S' can be written as

$$x^2 + y^2 + z^2 - c^2t^2 = 0 \quad \& \quad x'^2 + y'^2 + z'^2 - c^2t'^2 = 0$$

$$\Rightarrow x'^2 + y'^2 + z'^2 - c^2t'^2 = x^2 + y^2 + z^2 - c^2t^2 \quad \text{—— (1)}$$

From Lorentz transformation equations

$$y' = y, \quad z' = z$$

$$\Rightarrow x'^2 - c^2t'^2 = x^2 - c^2t^2 \quad \text{—— (2)}$$

Let $\quad x' = \alpha(x - vt) \quad$ —— (3)

& $\quad t' = \beta(t + \gamma x) \quad$ —— (4)

(3) (4) & (2) $\Rightarrow \alpha^2(x - vt)^2 - c^2\beta^2(t + \gamma x)^2 = x^2 - c^2t^2$

$\Rightarrow \alpha^2(x^2 + v^2t^2 - 2xvt) - \beta^2c^2(t^2 + \gamma^2x^2 + 2xt\gamma) = x^2 - c^2t^2$

comparing the co-efficients of x^2, t^2 & $2xt$ we get

$\alpha^2 - \beta^2c^2\gamma^2 = 1$ —— (5)

$\alpha^2v^2 - \beta^2c^2 = -c^2$ —— (6)

$\alpha^2 v + \beta^2c^2\gamma = 0$ —— (7) $\Rightarrow \gamma = -\dfrac{\alpha^2 v}{\beta^2c^2}$ —— (8)

(1) & (8) $\Rightarrow \alpha^2 - \beta^2c^2 \cdot \dfrac{\alpha^4v^2}{(\beta^2c^2)^2} = 1 \Rightarrow \alpha^2 - \dfrac{\alpha^4v^2}{\beta^2c^2} = 1$

$\Rightarrow \alpha^2 - 1 = \dfrac{\alpha^4v^2}{\beta^2c^2} \Rightarrow \beta^2c^2 = \dfrac{\alpha^4v^2}{\alpha^2 - 1}$ —— (9)

(6) & (9) $\Rightarrow$ $\alpha^2 v^2 - \frac{\alpha^4 v^2}{\alpha^2 - 1} = -c^2$

$\Rightarrow \cancel{\alpha^4 v^2} - \alpha^2 v^2 - \cancel{\alpha^4 v^2} = -\alpha^2 c^2 + c^2$

$\Rightarrow \alpha^2 (c^2 - v^2) = c^2$

$\alpha^2 = \frac{c^2}{c^2 - v^2}$ —(10) $\Rightarrow$ $\alpha = \frac{1}{\sqrt{1 - v^2/c^2}}$ —(11)

(5) & (10) $\Rightarrow$ $\beta^2 c^2 = \alpha^2 v^2 + c^2 = \frac{c^2 v^2}{c^2 - v^2} + c^2$

$\Rightarrow \beta^2 = \frac{v^2}{c^2 - v^2} + 1 = \frac{v^2 + c^2 - v^2}{c^2 - v^2}$

$\Rightarrow \beta^2 = \frac{c^2}{c^2 - v^2}$ —(12)

(10) & (12) $\Rightarrow$ $\alpha^2 = \beta^2 \Rightarrow \beta = \alpha = \frac{1}{\sqrt{1 - v^2/c^2}}$ —(13)

(13) & (8) $\Rightarrow$ $\gamma = -\frac{v}{c^2}$ —(14) $(\because \alpha^2 = \beta^2)$

Therefore Lorentz Transformation equations are

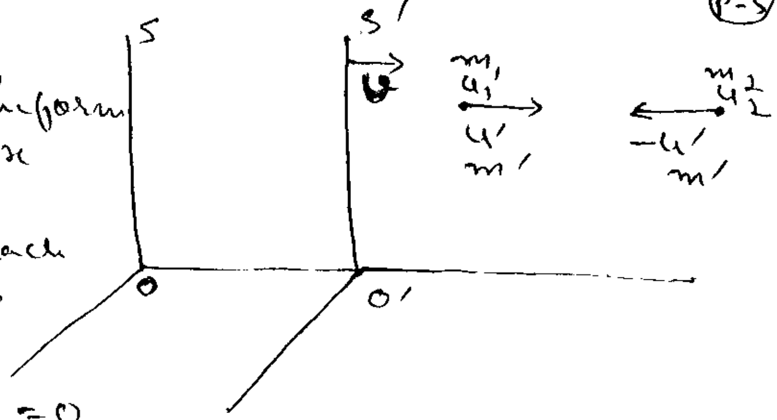
$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$

$y' = y$

$z' = z$

& $t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - v^2/c^2}}$

Q2 (c) S & S' are inertial frames of reference. S' moves with uniform velocity v w.r.t S in +ve x direction.



Let two identical particles each of mass m' w.r.t O' move with velocities u' & $-u'$

Their initial momentum $= 0$

$\therefore$ Momentum after collision will also be zero

For O m_1 & m_2 will be masses of two particles. Let particles coalesce into a single particle after collision so that velocity of combination $= v$ for O.

Applying law of conservation of momentum for O

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$

$$\Rightarrow m_1(u_1 - v) = m_2(v - u_2) \quad \text{--- (1)}$$

From addition of velocity

$$u_1 = \frac{u' + v}{1 + \frac{u'v}{c^2}} \quad \text{--- (2)} \qquad \& \qquad u_2 = \frac{-u' + v}{1 - \frac{u'v}{c^2}} \quad \text{--- (3)}$$

(1) (2) & (3) $\Rightarrow$

$$m_1\left[\frac{u'+v}{1+\frac{u'v}{c^2}} - v\right] = m_2\left[v - \frac{v-u'}{1-\frac{u'v}{c^2}}\right]$$

$$m_1\left[\frac{u' + \cancel{v} - \cancel{v} - \frac{u'v^2}{c^2}}{1 + \frac{u'v}{c^2}}\right] = m_2\left[\frac{\cancel{v} - \frac{u'v^2}{c^2} - \cancel{v} + u'}{1 - \frac{u'v}{c^2}}\right]$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{1 + \frac{u'v}{c^2}}{1 - \frac{u'v}{c^2}} \quad \text{--- (4)}$$

Also

$$1 - \frac{u_1^2}{c^2} = 1 - \frac{u'^2 + v^2 + 2u'v}{\left(1 + \frac{u'v}{c^2}\right)^2 c^2}$$

$$= \frac{c^2 + \frac{u'^2 v^2}{c^2} + \cancel{2u'v} - u'^2 - v^2 - \cancel{2u'v}}{\left(1 + \frac{u'v}{c^2}\right)^2 c^2}$$

$$1 - \frac{u_1^2}{c^2} = \frac{c^2 + \frac{u'^2 v^2}{c^2} - u'^2 - v^2}{\left(1 + \frac{u'v}{c^2}\right)^2 c^2} \quad \text{--- (5)}$$

Similarly for 2nd particle, replacing u' by $-u'$

$$1 - \frac{u_2^2}{c^2} = \frac{c^2 + \frac{u'^2 v^2}{c^2} - u'^2 - v^2}{\left(1 - \frac{u'v}{c^2}\right)^2 c^2} \quad \text{--- (6)}$$

(6)/(5) $\Rightarrow$

$$\frac{1-\frac{u_2^2}{c^2}}{1-\frac{u_1^2}{c^2}} = \frac{\left(1+\frac{u'u}{c^2}\right)^2}{\left(1-\frac{u'u}{c^2}\right)^2} \quad \text{--- (7)}$$

(4) & (7) $\Rightarrow$

$$\frac{1-\frac{u_2^2}{c^2}}{1-\frac{u_1^2}{c^2}} = \left(\frac{m_1}{m_2}\right)^2$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{\sqrt{1-\frac{u_2^2}{c^2}}}{\sqrt{1-\frac{u_1^2}{c^2}}}$$

If velocity u' is so chosen that $u_2 = 0$ then $m_2 = m_0$ (rest mass)

$$\Rightarrow \quad \frac{m_1}{m_0} = \frac{1}{\sqrt{1-\frac{u_1^2}{c^2}}}$$

m_1 is mass corresponding to velocity u_1. Therefore, if m is mass corresponding to velocity u then $u_1 \to u$ & $m_1 \to m$

$$\Rightarrow \quad \frac{m}{m_0} = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$m = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}} \quad \text{------} \quad (8)$$

<u>This shows that mass depends upon velocity</u>

case I when $u \lll c$ $\quad m \simeq m_0$

case ii) when $u = c$ $\quad m = \infty$

case iii) when $u > c$ $\quad m$ is imaginary

This shows that, it is impossible for any particle to have ~~....~~ velocity larger than velocity of light

53
c

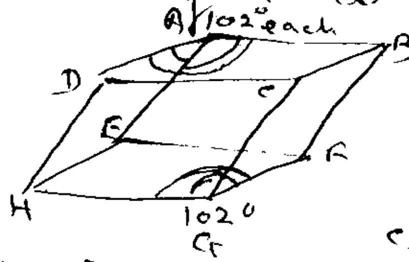
There are certain crystals on which if unpolarised lights is allowed to fall, then the light is bend in two directions instead of one direction. This phenomenon of bending of light in two directions instead of one direction while passing through special crystals like calcite or quartz crystals is called double refraction

Ray of light is split into two rays called
i) ordinary rays which have vibrations ⊥ to principal section
ii) Extraordinary rays which as vibrations in principal section of crystal.

If light from a pt source falls normally on the crystal, then two images are formed. If crystal is rotated, then one image remains stationary (corresponding to ordinary ray) and other rotates (This corresponds to extra ordinary ray)

Ordinary ray has same velocity in all directions whereas extra ordinary ray has different velocities in different directions

Both ordinary and extra ordinary rays have same velocity along optic axis



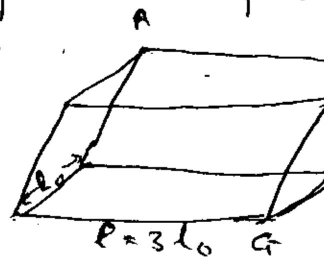
In calcite crystal there are two corners A & G where all the 3 angles are 102°. These corners are called Blunt corners

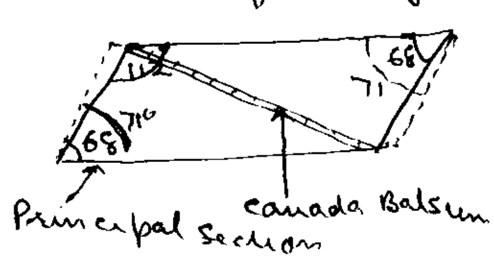
A line passing through any blunt corner and equally inclined to 3 faces is called optic axis. Any other line ∥ to optic axis is also optic axis.

Planes containing optic axis and ⊥ to any pair of opposite faces is called principle section.

Vibrations of O-ray are ⊥ to p-section & vib of E-ray are ∥ to p-section

Nicol Prism





Construction: Nicol prism is a calcite crystal with its length 3 times its breadth. It is cut parallel to its principal section having adjacent angle 109° and 71°.

Its end faces are grounded in such a way that adjacent angles are 112° and 68°.

Crystal is cut through one blunt corner to another along a plane ⊥ to principal section.

Cut faces are grounded, polished and joined to each other with Canada Balsam (a glass cement) having refractive index midway between those for ordinary & extra ordinary ray

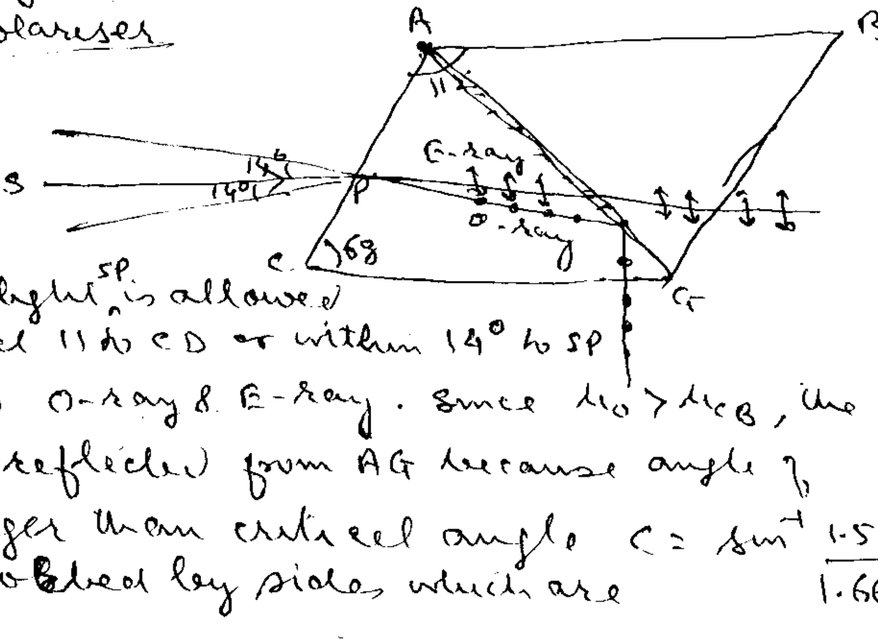
$\mu_o = 1.66$

$\mu_{CB} = 1.55$

$\mu_e = 1.49$ ⊥ to p-section to 1.66 along p axis.

Working: Nicol prism can be used as i) polariser & ii) Analyser

Nicol prism as polariser



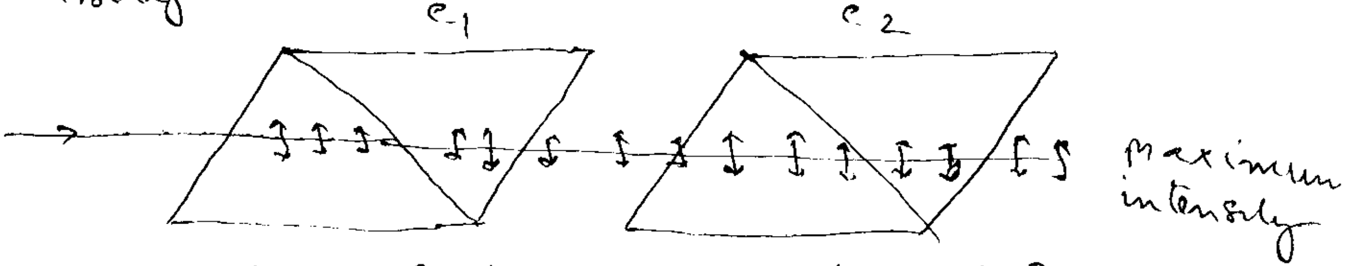
when unpolarised light SP is allowed to fall on crystal ∥ to CD or within 14° to SP it is splitted into O-ray & E-ray. Since $\mu_o > \mu_{CB}$, the O-ray is totally reflected from AG because angle of incidence is larger than critical angle $C = \sin^{-1}\frac{1.55}{1.66} = 69°$

O-rays are also absorbed by sides which are blackened.

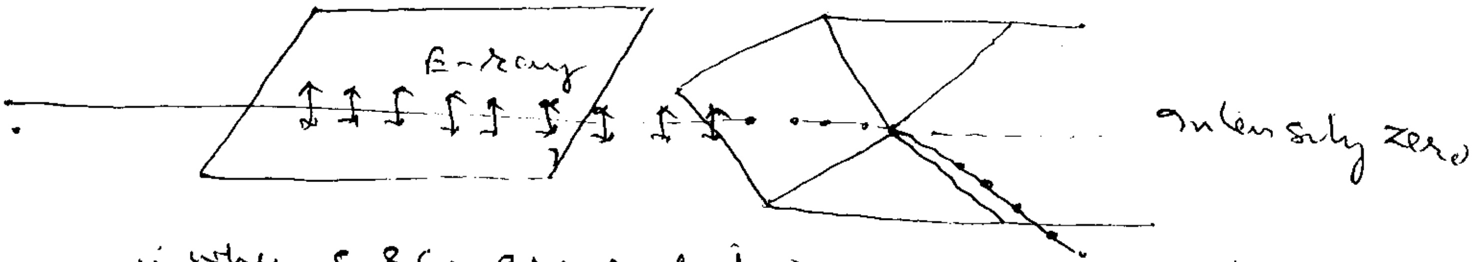
E-rays having vibrations parallel to p-section pass through crystal and hence are plane polarised. Thus Nicol prism acts as polariser

Nicol prism as analyser

If we take two Nicol prisms with their p-sections ∥ to each other, then E-rays coming out of first crystal with vibrations ∥ to p-section pass through 2nd Nicol prism as such giving maximum intensity



However when p-sections of crystals C_1 & C_2 are in <u>crossed positions</u> (i.e ⊥ to each other) the E-ray coming out of C_1 becomes O-ray for C_2 due to vibrations ⊥ to p-section of C_2. The ray is therefore, totally reflected making the out coming intensity zero.

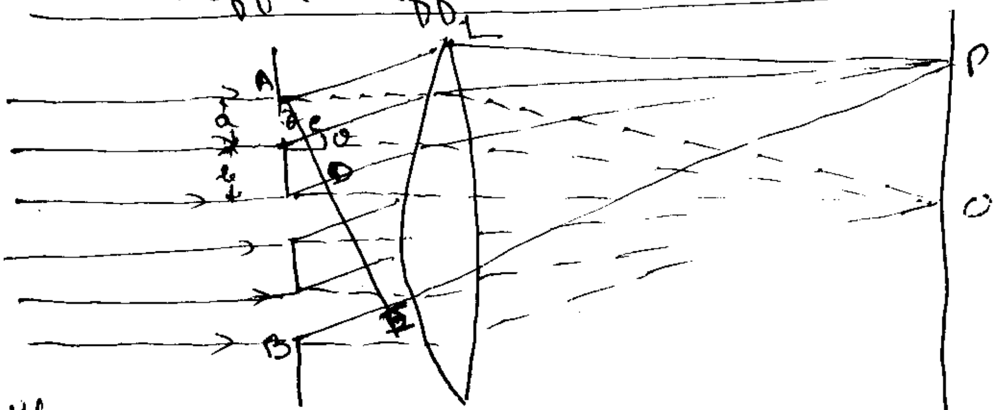


∴ when C_1 & C_2 are rotated w.r.t each other the intensity changes from maximum (when p-section are ∥) to zero (when p-sections are ⊥)

Thus Nicol prism acts as analyser.

Q4
c:

Fraunhoffer diffraction due to N-slits



Plane wave front is allowed to fall normally on a diffraction grating containing N 11 slits.

Undiffracted rays meet at O through a lens at the focus of lens.

Rays diffracted at angle θ meet at P producing principle maximas & minimas depending upon path difference between rays.

From each slit infinite no of rays interfere with each other to produce diffraction pattern

If 2α is phase difference between extreme rays of each slit & R' is amplitude of resultant then

$$2\alpha = \frac{2\pi}{\lambda} a \sin\theta \quad \text{—— (1)} \quad \& \quad R' = A_0 \frac{\sin\alpha}{\alpha} \quad \text{—— (2)}$$

a = width of slit. A_0 = Total amplitude of all the waves from each slit

$$\alpha = \frac{\pi}{\lambda} a \sin\theta \quad \text{—— (3)}$$

Due to N slits, N rays combine to produce diffraction pattern with the resultant R, given by

$$R = R' \frac{\sin N\beta}{\sin\beta}. \quad \text{—— (4)}$$

where $2\beta = \frac{2\pi}{\lambda}(a+b)\sin\theta$ is phase difference between two successive rays from two successive slits.

$$\Rightarrow \quad \beta = \frac{\pi}{\lambda}(a+b)\sin\theta \quad \text{—— (5)}$$

(2) & (4) $\Rightarrow$
$$R = A_0 \frac{\sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta} \quad \text{—— (6)}$$

Due to $\frac{\sin\alpha}{\alpha}$ term the successive diffraction maximas have intensities in the ratio of $1 : \frac{1}{21} : \frac{1}{63}$ ....

Principal Maxima

when $\beta = n\pi$ $N\beta = Nn\pi \Rightarrow \frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$ form

$$\Rightarrow \lim_{\beta \to n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to n\pi} \frac{N \cos N\beta}{\cos\beta} = N = \max.$$

$\Rightarrow$ Principal maximas are obtained for

$$\beta = \pm n\pi$$

$$\Rightarrow \frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi$$

$$\Rightarrow (a+b)\sin\theta = \pm n\lambda \quad \text{——— (7)}$$

Secondary minimas.

when $\sin N\beta = 0$ But $\sin\beta \neq 0$
then $R = 0$ & we get minima

$\Rightarrow$ for secondary minima
$$\sin N\beta = \sin n\pi$$

$$N\beta = n\pi \Rightarrow N\frac{\pi}{\lambda}(a+b)\sin\theta = n\pi$$

$$N(a+b)\sin\theta = n\lambda \quad \text{——— (8)}$$

n can have values $0, 1, 2 \ldots N-1, N+1, \cancel{N+2} \ldots 2N-1, \ldots$ $nN-1, nN+1 \ldots$

n can not have values $N, 2N, 3N \ldots$ because these values give conditions for maximas.

$\Rightarrow$ Between any two ~~secondary~~ principal maximas there are $N-1$ secondary minimas.

$\Rightarrow$ There must be $N-2$ secondary maximas also.

Condition for Missing order

From eq (2) when $\sin\alpha = 0$ but $\alpha \neq 0$ we get minima.

$\therefore$ condition for minima from each slit is

$$\sin\alpha = 0 = \sin m\pi$$

$$\Rightarrow \alpha = m\pi$$

$$\Rightarrow \frac{\pi}{\lambda} a\sin\theta = m\pi \Rightarrow a\sin\theta = m\lambda \quad \text{——— (9)}$$

Also for Principal maxima of grating

$$(a+b)\sin\theta = n\lambda \quad \text{——— (10)}$$

when both conditions (9) & (10) are satisfied simultaneously, the order of spectrum will be missing because waves from each slit cancel each other.

(10)/(9)
$$\frac{a+b}{a} = \frac{n}{m} \quad \text{——— (11)}$$

when $b = a$ $\quad n = 2m \Rightarrow n = 2, 4, 6, 8 \ldots$ will be missing

when $b = 2a$ $\quad n = 3m \quad n = 3, 6, 9 \ldots$ " "

and so on ......