Solution of Modal Paper

Subject - Physics - I

B. Tech First Semester - 2008 - 09

Section - A

Objective Type Questions:

Ques-1 (d) Either (a) & (b)

Ques-2 (a) Ruby Laser

Ques-3 (a) Perpendicular

Ques-4 (b) Dark

Ques-5 (b) In the same state of Polarization

Ques-6 (b) Fresnel

Ques-7 (C) Sodium light

Ques-8 (C) Straight

Ques-9 (C) Quartz

Ques-10 (C) Critical Angle

Secton - B

Short Answer type questions :

Ques-1 B We Know that

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Here $v = \frac{c}{\sqrt{2}}$

So, $$m = \frac{m_0}{\sqrt{1-\frac{(c/\sqrt{2})^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{1}{2}}} = \sqrt{2}\, m_0$$

Momentum $P = mv = (\sqrt{2}\, m_0)\frac{c}{\sqrt{2}} = m_0 c$

$$P = m_0 c$$

• Total Energy $E = mc^2 = (\sqrt{2}\, m_0)c^2 = \sqrt{2}\, m_0 c^2$

$$E = \sqrt{2}\, m_0 c^2$$

Kinetic Energy = Total Energy − Rest Energy

$= \sqrt{2}\, m_0 c^2 - m_0 c^2$

$= (\sqrt{2}-1)\, m_0 c^2 = (1.414-1)\, m_0 c^2$

Kinetic Energy $= 0.414\, m_0 c^2$

Ques-2 B From velocity addition Theorem

$$u = \frac{u'+v}{1+\frac{u'v}{c^2}}$$

$u' = 0.7c$ and $v = 0.7c$

$$u = \frac{0.7c + 0.7c}{1+\frac{(0.7c)(0.7c)}{c^2}} = 0.939c$$

$$u = 0.939c$$

Ques-3 B

Here $n = 4$, $(a+b) = \frac{1}{5000}$ cm

we know that -

$$(a+b)\sin\theta = n\lambda$$

For the longest wavelength, $\sin\theta = 1$

$$\lambda = \frac{a+b}{n} = \frac{1}{5000\times 4}$$

$$\lambda = 5\times 10^{-5}\text{ cm} = 5000\times 10^{-8}\text{ cm}$$

The longest wavelength $= 5000$ A°

Ques-4 B

The thickness of quarter wave Plate

$$t = \frac{\lambda}{4(\mu_e - \mu_o)}$$

$\mu_o = 1.54425$, $\mu_e = 1.55336$

$$\lambda = 5.893\times 10^{-5}\text{ cm}$$

$$t = \frac{5.893\times 10^{-5}}{4(1.55336 - 1.54425)} = 1.62\times 10^{-3}\text{ cm}$$

$$t = 1.62\times 10^{-3}\text{ cm}$$

Ques-5 B

The specific rotation S, corresponding to an optical rotation θ is given by

$$S = \frac{\theta}{l\times C}$$

$$C = \frac{\theta}{l\times S}$$

$\theta = 11°$, $l = 200$ mm $= 20$ cm $= 2$ dm, $S = 66°$

$$C = \frac{11}{2\times 66} = \frac{1}{12} = 0.0833\text{ gm/cc}$$

$$C = 0.0833\text{ gm/cc}$$

Ques-6 B $n_1 = 1.45$ and $n_2 = 1.40$

$$NA = \sqrt{(1.45)^2 - (1.40)^2} = \sqrt{2.1025 - 1.96}$$

$$NA = \sqrt{0.1425} = .3774$$

$$NA = \sin \phi_o$$

$$\phi_o = \sin^{-1}(NA)$$

$$\phi_o = \sin^{-1}(.3774) = 22.14$$

$$\phi_o = 22^\circ$$

Ques-7 B using the formula

$$\lambda = \frac{D^2_{n+p} - D^2_n}{4PR}$$

$D_n = 4.2mm = 0.42$ cm

$D_{n+p} = D_{n+14} = 7.0$ mm $= 0.70$ cm

$P = n + 14 - n = 14$

$R = 1m = 100$ cm

$$\lambda = \frac{(0.70)^2 - (0.42)^2}{4 \times 14 \times 100} = \frac{0.49 - 0.1764}{5600} \text{ cm}$$

$$\lambda = 5.6 \times 10^{-5} \text{ cm} = 5600 \, A^\circ$$

$$\lambda = 5600 \, A^\circ$$

Section - C

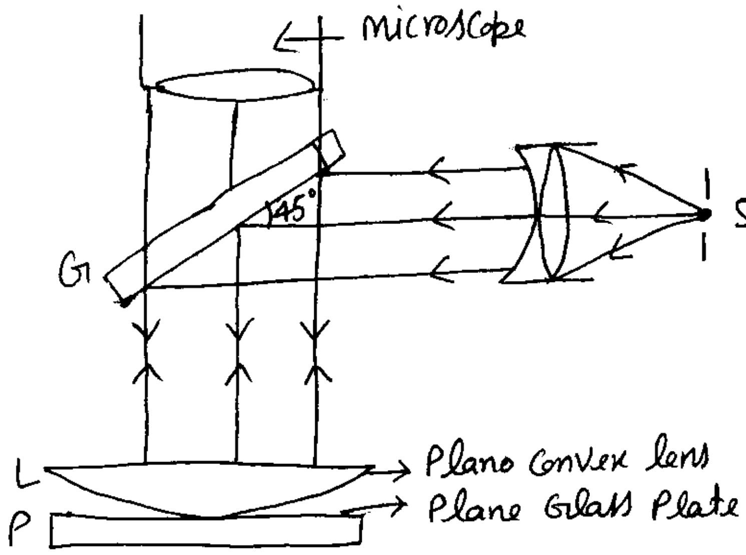
Long Answer Type Questions:

Ques-1 c

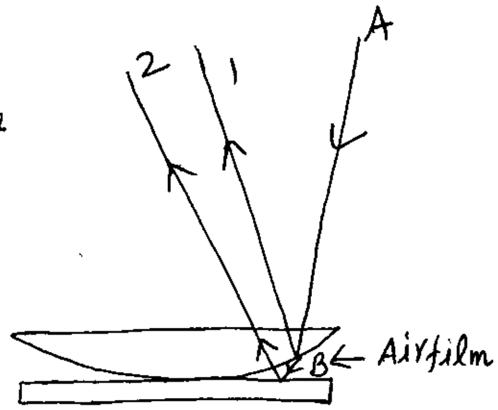
Formation of Newton's Rings →

When a plano-convex lens of large radius of curvature is placed on a plane glass plate, an air film is formed between the lower surface of lens and upper surface of the plate. The thickness of film is increases gradually from the paint of contact outwards. When monochromatic light is allowed to fall normally on this film, a system of alternate bright and dark concentric rings, with their centre dark, is formed in the air film. These rings were first developed by Newton and hence are known as Newton's rings.

(a)



(b)



Newton's rings are formed as a result of interference between the waves reflected from the top and bottom faces of the air film formed between the lens and plate.

From fig. (b), AB be a beam of monochromatic light of wavelength λ incident on the film. As a result of reflection at the top and bottom faces of the film, ray 1 and 2 are the coherent rays which interfere in the reflected system. For constructive interference, the path difference between them should be

$$2\mu t \cos r + \frac{\lambda}{2}$$

For normal incidence $r = 0$ and air film $\mu = 1$,

So, Path difference, $\Delta = 2t + \frac{\lambda}{2}$

- At the point of contact $t = 0$

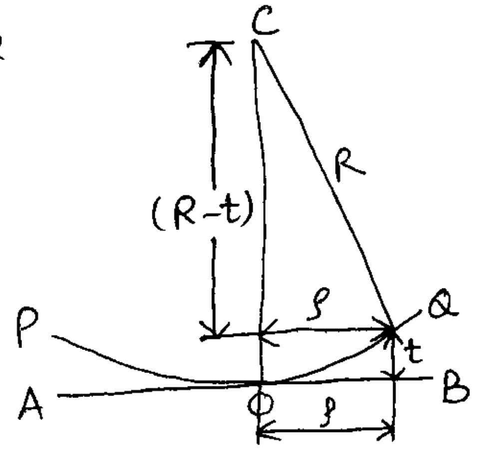
So, $\Delta = \frac{\lambda}{2}$

This is the condition of minimum intensity. Hence the centre of Newton's ring is dark.

<u>Diameters of dark and bright rings →</u>

Let POQ be a plano-convex lens placed on a plane glass plate AB. Let R be the radius of curvature of the lens surface in contact with the plate.

Let ρ be the radius of a Newton's ring corresponding to the constant film thickness 't'.



The path difference between two interfering rays

$$\Delta = 2t + \frac{\lambda}{2} \qquad - (1)$$

From figure -

$$R^2 = \rho^2 + (R-t)^2$$

or $$\rho^2 = R^2 - (R-t)^2$$

or $$\rho^2 = 2Rt - t^2$$

$t << R$

$$\rho^2 = 2Rt$$

$$\therefore 2t = \frac{\rho^2}{R} \qquad - (2)$$

So, the path difference between the interfering rays is $\frac{\rho^2}{R} + \frac{\lambda}{2}$.

<u>For dark rings</u> → The condition for dark rings

$$\text{Path difference} = \frac{\rho^2}{R} + \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$\frac{\rho^2}{R} = n\lambda \qquad \text{where } n = 0, 1, 2, 3 \ldots$$

If D is the diameter of Newton's ring,

then $\rho = \frac{D}{2}$

$$\therefore \frac{D_n^2}{4R} = n\lambda$$

$$D_n^2 = 4n\lambda R$$

$$D_n = \sqrt{4n\lambda R} \qquad - (3)$$

$$\boxed{D_n \propto \sqrt{n}}$$

Hence the diameters of the dark rings are proportional to the square roots of natural numbers.

For bright rings → The condition to get bright rings the path difference

$$\frac{\rho^2}{R} + \frac{\lambda}{2} = n\lambda$$

or

$$\frac{\rho^2}{R} = (2n-1)\frac{\lambda}{2} \qquad n = 1, 2, 3 \ldots$$

$$\boxed{\rho^2 = (2n-1)\frac{\lambda R}{2}}$$

Put $\rho = \frac{D}{2}$

$$\frac{D_n^2}{4} = (2n-1)\frac{\lambda R}{2}$$

$$D_n^2 = 2\lambda R(2n-1)$$

$$D_n = \sqrt{(2n-1)}\sqrt{2\lambda R}$$

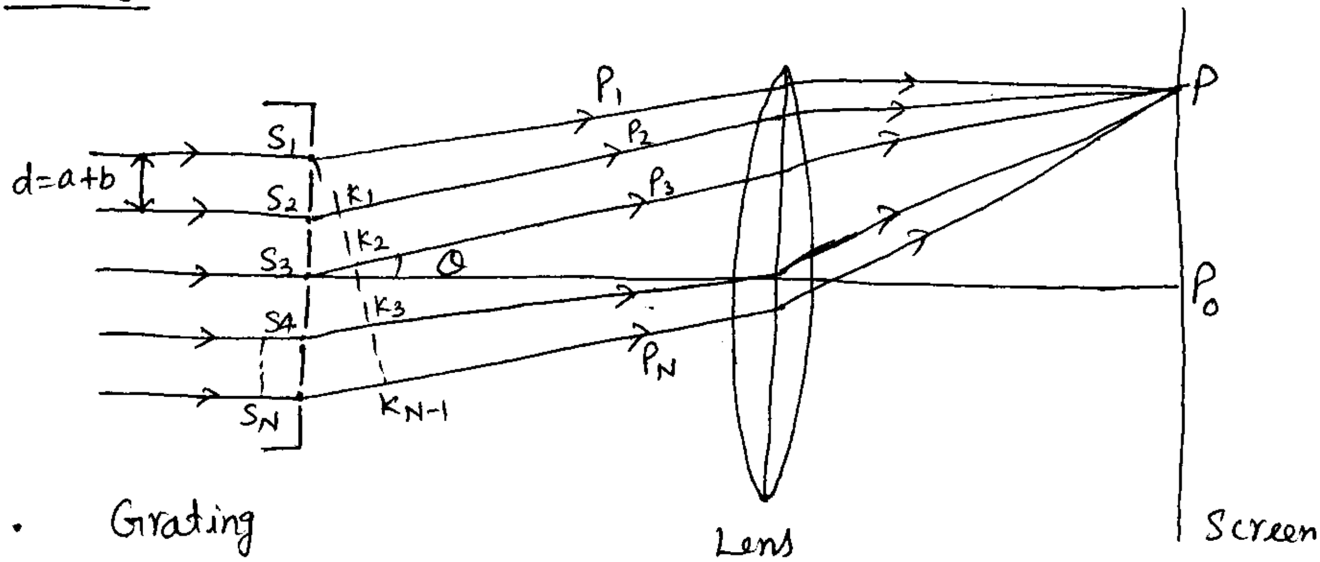
$$\boxed{D_n \propto \sqrt{(2n-1)}} \qquad n = 1, 2, 3 \ldots$$

Hence the diameters of the bright rings are proportional to the square root of odd natural numbers.

Ques-2 c Diffraction Grating → It is an arrangement consisting of a large number of parallel slits of same width and separated by equal opaque spaces. It is obtained by ruling equidistant parallel lines on a glass plate with the help of fine diamond paint. The lines act as opaque spaces and incident light cannot pass through them. The space between the two lines is ~~trans~~

transparent to light and acts as a slit. The number of lines in a plane transmission grating is of the order of 15,000 to 20,000 per inch.

Theory →



Let N be the number of parallel slits each of width 'a' and separated by opaque space 'b'. Then the distance between the centres of the adjacent slits is $d = (a+b)$ and is known as grating element.

The diffracted rays from each of the slits are allowed to fall on a convex lens which focus all of them at a point P on the screen.

As a single slit, the waves diffracted from each slit are equivalent to a single wave of amplitude

$$R = \frac{A \sin\alpha}{\alpha} \qquad -(1)$$

where

$$\alpha = \frac{\pi e \sin\theta}{\lambda} \qquad -(2)$$

The path difference between any two consecutive waves from two slits $(e+d)\sin\theta$. Therefore, the corresponding phase difference will be $\frac{2\pi}{\lambda}(e+d)\sin\theta$. Since the phase difference between any two consecutive waves given by

$$\frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta \quad - \quad (3)$$

The resultant of N- vibrations in a direction θ and each vibration is of amplitude $\left(\frac{A\sin\alpha}{\alpha}\right)$.

Now, similar to the resultant of n-harmonic waves, the resultant of N- slits may be

$$R' = \frac{R\sin\frac{2N\beta}{2}}{\sin\frac{2\beta}{2}} = \frac{R\sin N\beta}{\sin\beta}$$

$$R' = \frac{A\sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta}$$

Thus the resultant intensity at P may be given as

$$I = R'^2 = \frac{A^2\sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta} \quad - \quad (4)$$

Here the factor $\frac{A^2\sin^2\alpha}{\alpha^2}$ is the intensity factor due to single slit while $\frac{\sin^2 N\beta}{\sin^2\beta}$ is due to the interference from all the N- slits.

Principal Maxima → For maximum intensity

$$\sin\beta = 0 \quad \text{or} \quad \beta = \pm n\pi, \quad n = 0, 1, 2 \ldots$$

But in this condition $\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$ is in indeterminate form. Thus to find its value, we use L, Hospital rule

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to \pm n\pi} \frac{N \cos N\beta}{\cos\beta} = N$$

Thus $$I = \frac{A^2 \sin^2\alpha}{\alpha^2} N^2 \quad - (5)$$

The intensity of these maxima is maximum, so it is called principal maxima.

The condition of principal maxima is

$$\sin\beta = 0 \quad \text{or} \quad \beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

$$(e+d)\sin\theta = \pm n\lambda \quad - (6)$$

Now, for $n=0$, we get $\theta = 0$, which gives the direction of zero order principal maximum. The values $n = 1, 2, 3 ---$ correspond to the first, second --- order principal maxima. Here $\pm$ sign shows that the two principal maxima of the same order lie on either side of zero order maximum.

<u>Minima</u> → For $\sin N\beta = 0$, But $\sin\beta \neq 0$

So for minimum Intensity

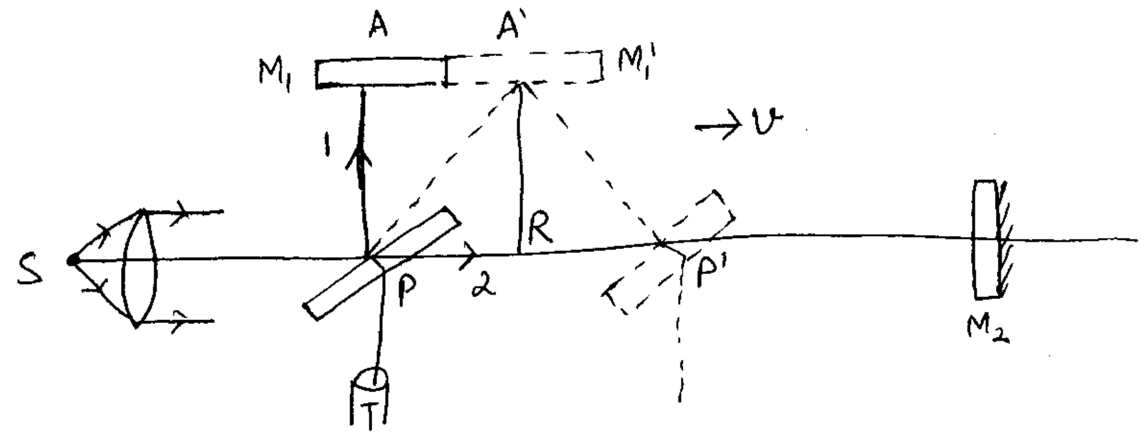
$$N\beta = \pm m\pi \quad \text{or} \quad \beta = \pm \frac{m\pi}{N}$$

$$N \cdot \frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi$$

$$N(e+d)\sin\theta = \pm m\lambda \quad - (7)$$

Thus for all integral values of m except $0, N, 2N --$ we get minima, because for $m = 0, N, 2N --$ the value of $\sin\beta = 0$ and this will give position of principal maxima.

Ques-3 C **Michelson - Morley Experiment** -



To detect the relative motion between ether and earth, Michelson performed an experiment in 1887 using Michelson interferometer.

- In this experiment light from the source S is incident upon a half-silvered glass plate P placed at 45° to the beam. It splits into beams 1 (reflected) and 2 (refracted). These beams travel right angles to each other. They incident normally on mirrors M_1 and M_2 and reflected back to P. The two beams returned to P are directed towards a telescope T and interference takes place. The interference fringes are observed in the telescope.

Suppose C is the velocity of light through the ether. The beam 2 moving towards M_2 has a velocity $(C-v)$ relative to the apparatus on the outgoing journey and $(C+v)$ on the return journey.

Let t_2 be the time required by ray 2 for the round trip journey, then

$$t_2 = \frac{d}{c-v} + \frac{d}{c+v} = \frac{2dc}{c^2-v^2}$$

$$t_2 = \frac{2d}{c}\left(1+\frac{v^2}{c^2}\right) \quad — (1)$$

Let t_1 be the time required by ray 1 for the round trip journey. Then $t_1 = 2t$

t = Time required to travel distance PA'

$AA' = vt$, $A'P' = ct$, $PA' = ct$, $A'R = d$

From $\Delta PA'R$

$$c^2t^2 = d^2 + v^2t^2$$

$$t^2 = \frac{d^2}{c^2-v^2}$$

$$t = \frac{d}{\sqrt{c^2-v^2}} = \frac{d}{c}\left(1+\frac{v^2}{2c^2}\right)$$

$$t_1 = 2t = \frac{2d}{c}\left(1+\frac{v^2}{2c^2}\right) \quad — (2)$$

The difference in times of travel of the two rays for their round trips is

$$\Delta t = (t_2 - t_1) = \frac{2d}{c} \cdot \frac{v^2}{2c^2} = \frac{dv^2}{c^3} \quad — (3)$$

The path difference introduced between the components of beam 1 and 2 will be, therefore

$$\delta = c \times \Delta t = \frac{2dv^2}{c^2} \quad — (4)$$

The number of fringes passing through a reference mark will be

$$N = \frac{\text{Path difference}}{\lambda} = \frac{2dv^2}{c^2\lambda} \quad — (5)$$

In the actual experiment performed by Michelson-Morley, $d = 11$ metres, $\lambda = 5.9 \times 10^{-7}$ m

$$c = 3\times10^{8}\ m/sec \quad v = 3\times10^{4}\ m/sec$$

$$N = 0.37$$

A fringe shift of this amount is readily detected with the apparatus. Michelson and Morley could not detect any shift in the fringes, when interferometer was rotated through 90°. This indicates that relative velocity between the earth and ether is zero.

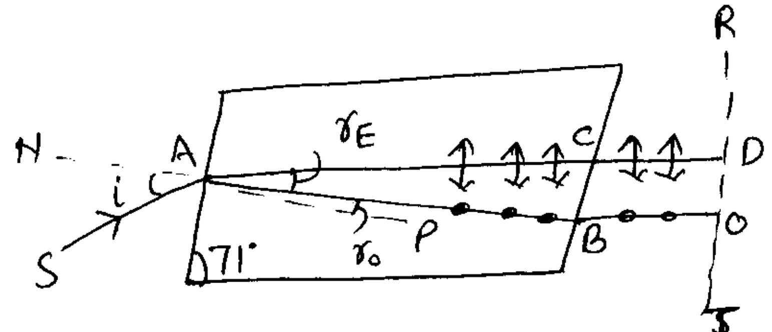
Explanation of the Negative Results →

1- The moving earth drags the ether with it. Hence there is no relative motion between the two so that no shift is observed.

2- Einstein proposed that the speed of light is constant in vacuum in all inertial frames and is independent of the motion of the source, observer and medium. It is known as principle of constancy of speed of light.

3- According to length contraction hypothesis the moving body contracted in the direction of motion relative to stationary ether by a factor $\sqrt{1-\frac{v^2}{c^2}}$. This contraction equalises the two times t_1 and t_2, hence no fringe shift is expected.

Ques.4 c **Double Refraction** → When ordinary light incident on a doubly refracting crystal, it splits up in O-and E-rays. O-ray follows ordinary laws of refraction while E-ray behaves in an extraordinary manner and does not obey the refraction laws. Therefore if an object is seen through such crystal its two images are formed, one due to O-ray and another due to E-ray. This phenomenon is known as double refraction.



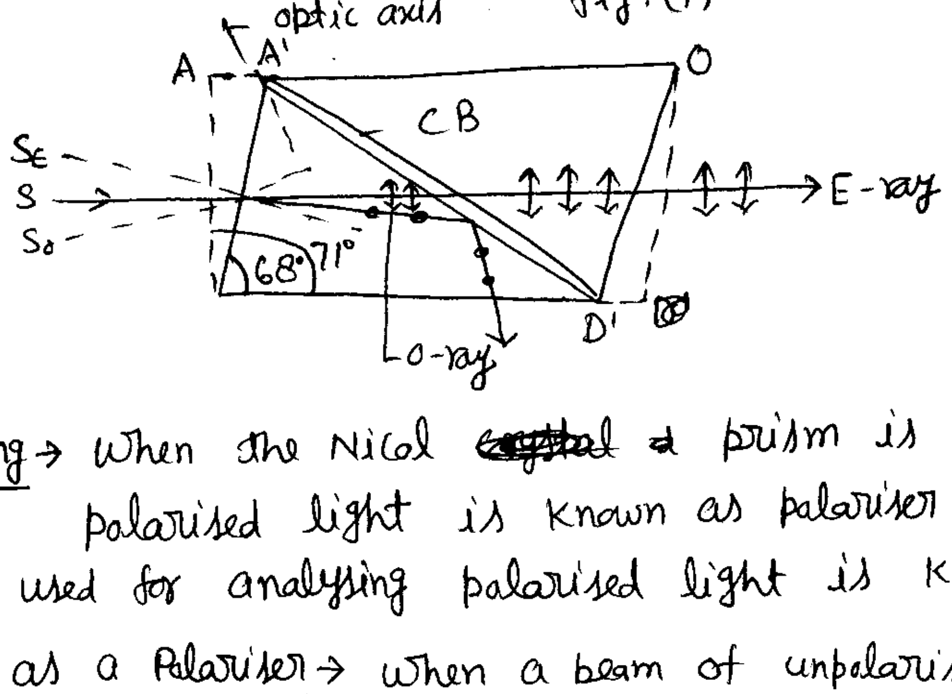
Let a ray SA of light incident on a calcite crystal at an angle of incidence is i. Let r_o and r_E be the angles of refraction of ordinary and extraordinary rays. The O- and E-rays emerges out from the crystal parallel to each other. Thus on a screen RT two images can be obtained.

For ordinary ray, refractive index $\mu_o = \frac{\sin i}{\sin r_o}$ remains constant, while for E-ray, $\mu_E = \frac{\sin i}{\sin r_E}$ varies with the angle of incidence.

Nicol Prism → Nicol prism is an optical device used for producing and analysing the plane polarised light. It works on the phenomenon of double refraction.

Construction → Nicol prism is commonly used to get a plane polarised light. It is made from calcite crystal. The length of the calcite crystal taken as three times its width. The end faces in the crystal are cut in such a manner that the acute angle (71°) of the principle plane reduces to 68°. It is cut into two parts along a plane which passes through the blunt corners A' and D' and is perpendicular to the principal section. Now the cut surfaces are polished and cemented back with a transparent substance known as Canada balsam. The refractive index of Canada balsam is midway for O-ray and E-ray.

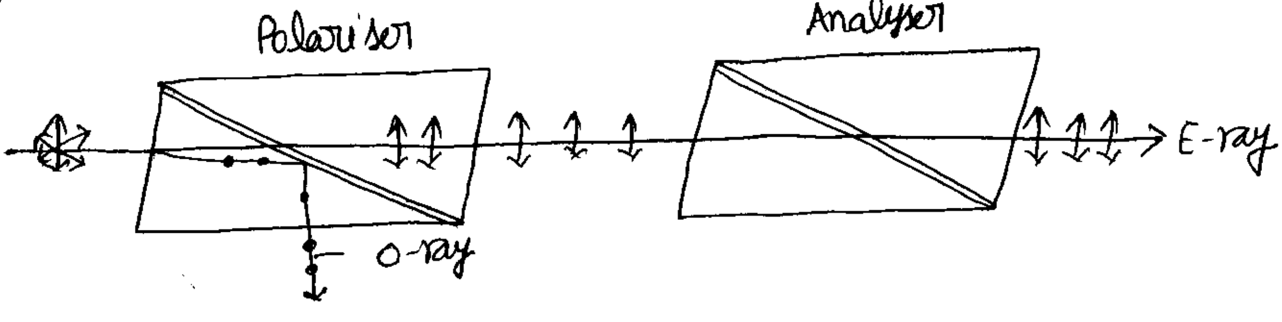
fig. (1)

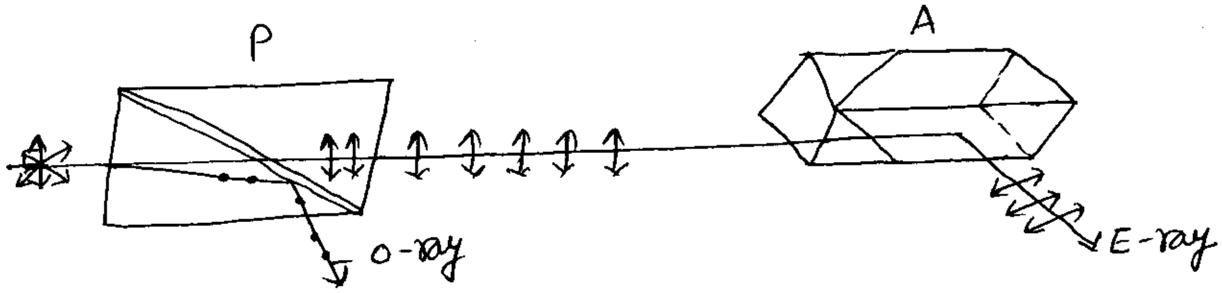


Working → When the Nicol ~~crystal~~ prism is used for producing polarised light is known as polariser and when it is used for analysing polarised light is known as analyser.

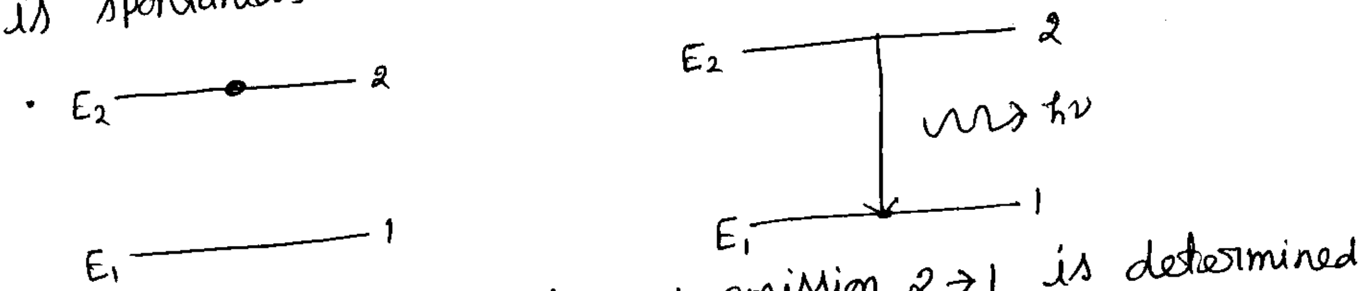
Nicol as a Polariser → When a beam of unpolarised light falls on the face A'P, splits into O- and E-rays. Now after traversing some distances into nicol both the rays reach to the Canada balsam layer. Since this layer offers a rarer medium for O-ray coming from a denser medium of calcite, the O-ray suffers total internal reflection provided that the angle of incidence of O-ray at Canada balsam layer is greater than the critical angle. The value of critical angle for O-ray is $\sin^{-1}\left(\frac{1.55}{1.68}\right) = 69°$. This totally reflected ray is absorbed by the crystal. The E-ray travels from rarer calcite medium to denser Canada balsam layer, therefore, it emerges out of the crystal.

Nicol as analyser → when the two nicol prisms are arranged coaxially then the first nicol acts a polariser and produced plane polarised E-ray. The E-ray emerging from the polariser falls on the second nicol prism known as analyser. When the the principal sections of the two nicol are parallel to each other, the plane polarised ray (E-ray) coming out from the first nicol is easily transmitted through the second nicol. Now if we ratate the second nicol gradually, the intensity of the transmitted ray from the second nicol decreases. The intensity decreases to zero when the principal sections of the two nicols become perpendicular to each other. In this position the two nicols are said to be crossed. Hence the second nicol works as an analyser.





Ques-5 (c) **Spontaneous Emission →** An atom initially in the higher state 2, Excited state with higher energy is unstable, hence atom in excited state does not stay for longer time and it jumps to the lower energy state 1 emitting a photon of frequency ν. This is spontaneous emission of radiation.

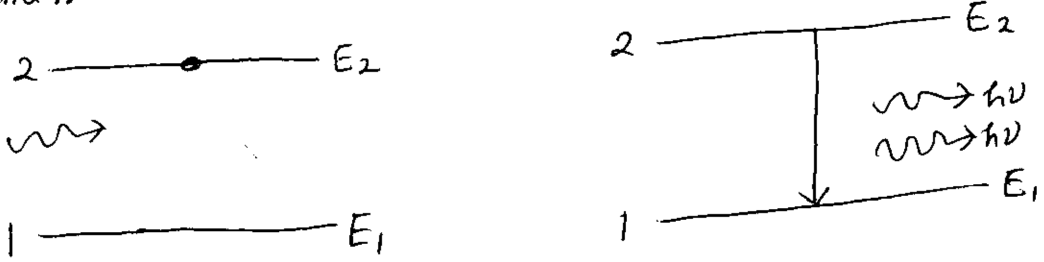


The probability of spontaneous emission $2 \to 1$ is determined by the properties of states 2 and 1.

$$P_{21} = A_{21}$$

A_{21} is the Einstein's Coefficient of spontaneous emission of radiation.

Stimulated Emission → An atom in an excited energy state may, under the influence of the e.m field of a photon of frequency ν incident upon it, jump to a lower energy state, emitting an additional photon of same frequency ν. This emission is known stimulated emission of radiation.



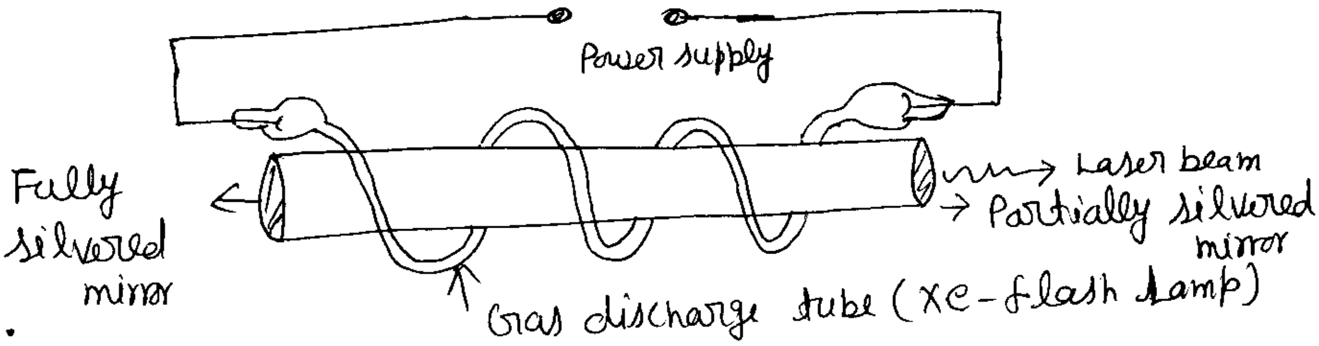
Ruby Laser → It has three energy levels of population inversion, i.e. it contains excited energy level(E_3), upper lasing level is the metastable state (E_2) and the lower lasing level is the ground state (E_1). It consists of following three parts -

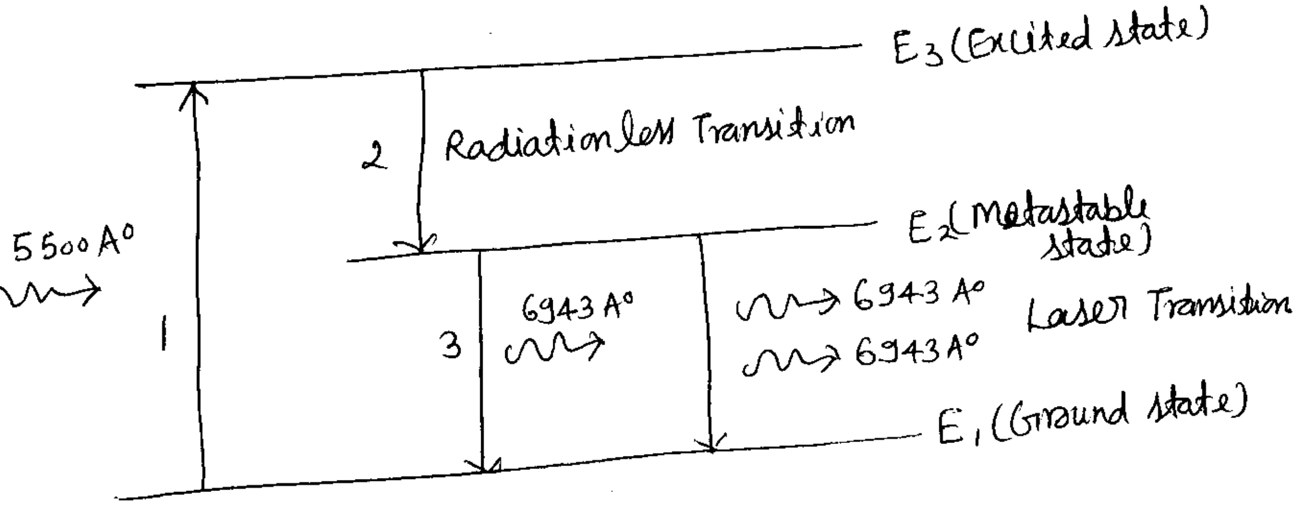
1- The working system is in the form of a rod of ruby crystal Al_2O_3.
2- The optical pumping system consisting of helical xenon discharge tube.
3- The resonant cavity is in the form of cylinderical tube which consist of two parallel reflecting plates (mirrors) at the two ends. The plate at the left end is fully silvered while at the right end is partially silvered.

The ruby rod is made up of aluminium oxide crystal doped with Cr_2O_3. This impurity of Cr^{3+} ions is responsible for the pink colour of ruby cylinderical rod. The length of the rod is 4 cm and diameter is 1.0 cm.

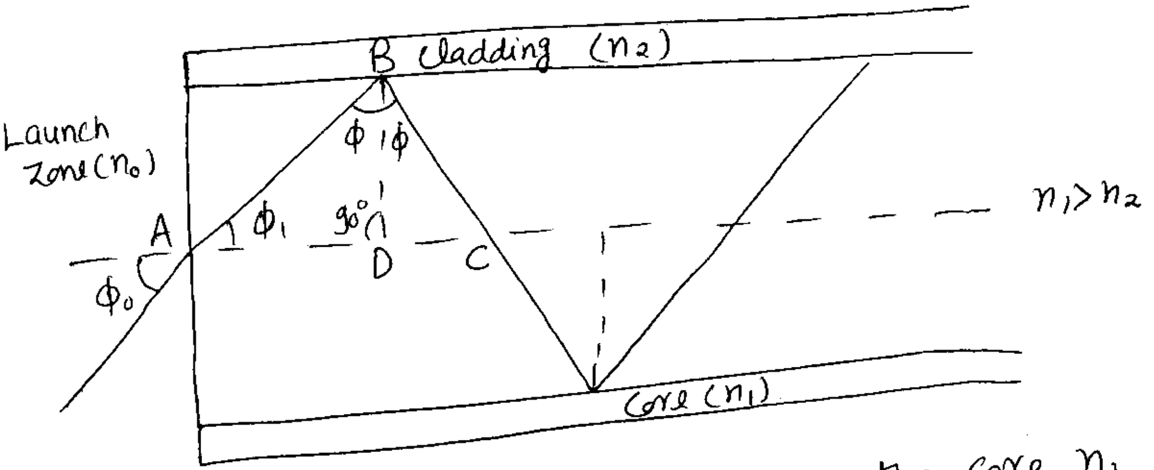
Working → The life time of metastable state of chromium ions (Cr^{3+}) is about 3×10^{-3} sec. In this laser the energy source used is xenon flash lamp. Thus xenon flash lamp excites the Cr^{3+} ions from ground state to higher energy levels. These ions stay in higher energy level only for a very short time interval and jump to a lower metastable state by losing energy to other ions in the crystal. Hence the transition from $E_3 \rightarrow E_2$ is a radiationless transition.

Since the metastable state E_2 is a long lived state (life time 10^{-3} sec), the number of Cr^{3+} ions goes on increasing in this state while due to pumping the number in the ground state goes on decreasing. Thus population inversion is achieved between metastable state and ground state. Some Cr^{3+} ions decay spontaneously from metastable state by emitting a photon. This photon move back and forth inside the ruby rode and excite other Cr^{3+} ions to radiate. Thus we get a amplified, intense laser beam of wavelength 6943 A°.





Ques-6 Acceptace angle is defined as the maximum external
c incidence angle for which the light will propagate in the fibre.



Consider a ray of light enters the core n_1 through the launch zone with an angle of incidence ϕ_0 and leaves the interface at an angle ϕ_1 which is smaller than angle of incidence. From Snell's Law, incidence angle ϕ_0 is related to the refraction angle ϕ_1 as

$$n_0 \sin \phi_0 = n_1 \sin \phi_1 \quad -(1)$$

From Δ ABD

$$\phi_1 + \phi + 90^\circ = 180^\circ$$

$$\phi_1 = 90^\circ - \phi \quad -(2)$$

using equ. (2) equ. (1) becomes

$$n_0 \sin \phi_0 = n_1 \sin (90^\circ - \phi)$$

$$n_0 \sin \phi_0 = n_1 \cos \phi$$

$$\sin \phi_0 = \frac{n_1}{n_0} \cos \phi \quad -(3)$$

To propagate within the fibre the internal reflection angle ϕ must be greater than critical angle ϕ_c

$$\sin \phi_c = \frac{n_2}{n_1}$$

$$\cos \phi_c = \sqrt{1 - \sin^2 \phi_c} = \sqrt{1 - \frac{n_2^2}{n_1^2}} = \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \quad -(4)$$

using equ.(4), equ.(3) becomes

$$\sin\phi_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

$$\boxed{\phi_0 = \sin^{-1}\frac{\sqrt{n_1^2 - n_2^2}}{n_0}}$$

Numerical Aperature → The numerical aperature is defined as the light gathering ability of the fibre.

The numerical aperature of the fibre is given as the sine of the maximum accepstance angle and numerical aperature is usually measured with air in front of the fibre i.e. $n_0 = 1$.

$$NA = \sin\phi_0\,(\max) \quad \text{with } n_0 = 1$$

$$NA = \sqrt{n_1^2 - n_2^2} \qquad \text{--- (1)}$$

The normalized difference Δ, used in the optical fibre which is defined as the ratio of difference between the refractive indices of the core and cladding to the refractive index of the core i.e.

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$\therefore \quad \Delta = 1 - \frac{n_2}{n_1}$$

$$\text{or} \quad \frac{n_2}{n_1} = 1 - \Delta \qquad \text{– (2)}$$

From equ. (1) $\quad NA = n_1\sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} \qquad$ – (3)

using equ. (2), equ. (3) becomes

$$NA = n_1\sqrt{1 - (1-\Delta)^2}$$

$$NA = n_1\sqrt{2\Delta - \Delta^2}$$

As the difference between the refactive indices of the core and cladding is very small, so Δ^2 is neglected

$$\boxed{NA = n_1\sqrt{2\Delta}}$$