[MODEL QUESTION PAPER]

B. Tech. I Sem-1

MATHEMATICS-I

Time - 3hr. Max. Marks: 100

NOTE :- The Question paper contains three sections. Section A, Section B & Section C with the weightage of 20, 30 & 50 marks respectively. Follow the instructions as given in each section.

SECION - A

Note → Attempt all questions.

Q-1 Let A & B any two matrices such that AB = 0 and A is non-singular then
(a) B = 0 (b) B is also non singular (c) B = A (d) B is singular

Q-2 If A is non-zero column vector (n×1); Then the rank of matrix AA^T is
(a) 0 (b) 1 (c) n-1 (d) n

Q-3 Inverse of $\begin{bmatrix} 4 & 3 \\ -7 & 1 \end{bmatrix}$ is
(a) $\begin{bmatrix} 1/4 & 1/3 \\ -1/7 & 1 \end{bmatrix}$ (b) $\frac{1}{25}\begin{bmatrix} 4 & 3 \\ -7 & 1 \end{bmatrix}$ (c) $\frac{1}{25}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. (d) $\frac{1}{25}\begin{bmatrix} 1 & -3 \\ 7 & 4 \end{bmatrix}$

Q-4 Two of given eigen values of a 3×3 matrix, whose determinant equals 4 are -1 & +2 the third eigen value of the matrix is equal to
(a) -2 (b) -1 (c) 1 (d) 2

Q-5 Fill in the blanks of the following statements:
(a) If $y = \frac{ax+b}{cx+d}$, then $2y_1y_3 =$ ____________
(b) $\frac{(-1)^{n-1}(n-1)!\,a^n}{(ax+b)^n}$ is the nth differential coefficient of ....
(c) If $x = u+v$; $y = u-v$, Then $\frac{\partial u}{\partial x} =$ .... & $\frac{\partial v}{\partial y} =$ ......

(d) $f(x,y) = f(2,3) + \ldots\ldots\ldots$?

Q-6 Write 'T' for true & 'F' for false statement

(a) If $u = \frac{y-x}{1+xy}$ & $v = \tan^{-1}y - \tan^{-1}x$ then $\frac{\partial(u,v)}{\partial(x,y)} = 0$

(b) If u, v, w of three independent variables x, y, z are not independent Then $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$

(c) Percentage error depends upon relative & absolute error.

(d) Conditions for $f(x,y)$ to the maximum are $f_x = f_y = 0$; $f_{xx}f_{yy} > f_{xy}^2$ & $f_{xx} > 0$

Q-7 Fill in the blanks

(a) $\int_1^0 \int_0^1 (x+y)\,dx\,dy = \ldots\ldots$

(b) The area of the cardiotol $r = a(1+\cos\theta)$ is ....

(c) $\Gamma\frac{1}{n}\,\Gamma\frac{2}{n}\,\Gamma\frac{3}{n} \ldots \Gamma\frac{n-1}{n} = \ldots\ldots$

(d) $\iiint_V x^{l-1} y^{m-1} z^{n-1}\,dx\,dy\,dz = \frac{\Gamma l\,\Gamma m\,\Gamma n}{\Gamma\ldots\ldots} \ldots\ldots$

Q-8 The magnitude of the vector drawn in a direction perpendicular to the surface $x^2+2y^2+z^2=7$ at the point $(1,-1,-2)$ is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) 6

Q-9 A necessary & sufficient condition that line integral $\int_C A\cdot dr = 0$ for every closed curve C is that the point (3,3,2) along the path C is

(a) div A = 0 (b) Curl A = 0 (c) div $A \neq 0$ (d) curl $A \neq 0$

Q-10 The value of the surface integral $\iint_S (yz\,dy\,dz + zx\,dz\,dx + xy\,dx\,dy)$. where S is the surface of the sphere $x^2+y^2+z^2=1$ is

(a) $4\pi/3$ (b) 0 (c) 4π (d) 12π

Q-11 If $\vec{f} = ax\vec{i} + by\vec{j} + cz\vec{k}$ a, b, c; constants, then $\iint f\,dS$ where S is the surface of a unit sphere, is

(a) 0 (b) $\frac{4}{3}\pi(a+b+c)$ (c) $\frac{4}{3}\pi(a+b+c)^2$ (d) none of these.

Section - B

Note :→ Attempt any Three questions. All questions carry equal marks : [10×3 = 30]

1. If $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$

Find the modal matrix P and the resulting diagonal matrix D of A.

2. If $y = e^{m \cos^{-1} x}$,

Prove that $(1-x^2) y_{n+2} - x(2n+1) y_{n+1} - (n^2+m^2) y_n = 0$

hence find the value of $(y_n)_0$

3. If $x = e^r \cos\theta$, $y = e^r \sin\theta$;

Prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = e^{-2r}\left[\dfrac{\partial^2 u}{\partial r^2} + \dfrac{\partial^2 u}{\partial \theta^2}\right]$

4. If u, v, w are the roots of the cubic equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ, Find $J(u,v,w)$

5. Verify divergence theorem for $\vec{F} = 4xz\,i - y^2 j + yz\,k$ taken over the cube bounded by the planes $x=0, x=1, y=1, z=0, z=1$.

6. The plane $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$ meets the axes in A, B and C. Apply Dirichlet's integral to find the volume of the tetrahedron $OABC$. Also find its mass if the density is $kxyz$.

SECTION - C

Attempt any two parts from each ~~section~~ question. All questions are compulsory. [10×5=20]

Q. 1. (a) If $x^x y^y z^z = c$ show that at
$x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log_e x)^{-1}$.

(b) If $u = \sec^{-1}\left(\frac{x^3 - y^3}{x+y}\right)$, show that
$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \cot u$.

(c) Trace the curve $x^3 + y^3 + 3axy$.

Q. 2. (a) Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

(b) Examine the extreme points of
$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

(c) Compute an approximate value of $(1.04)^{3.01}$

Q. 3. (a) Find the inverse of matrix A

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 0 \\ 1 & 4 & 1 \end{bmatrix}$$

(b) For what values of k, the equations $x+y+z=1$, $2x+y+4z=k$, $4x+y+10z=k^2$ have a solution and solve them completely in each case.

(e) Prove that the characterstic roots of a unitary matrix are of unit modulus.

Q.4. (a) Evaluate the following integral by changing the order of integration

$$\int_0^\infty \int_0^\infty \frac{e^{-y}}{y} \, dy \, dx$$

(b) Calculate the volume of the solid bounded by the surface $x=0$, $y=0$

$x+y+z=1$ & $z=0$

(c) Show that $\beta(p,q) = \int_0^1 \frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}}$

Q.5. (a) A vector field is given by

$\vec{A} = (x^2+xy^2)\hat{i} + (y^2+x^2y)\hat{j}$. Show that field is irrotational.

Find the scalar potential.

(b) Prove that $\operatorname{div}(\vec{a}\times\vec{b}) = \vec{b}\times \operatorname{Curl}\vec{a} - \vec{a}\times\operatorname{curl}\vec{b}$.

(c) Verify stokes theorem for the function $\vec{F} = x^2\hat{i} + xy\hat{j}$ integrated round the square whose sides are $x=0$, $y=0$, $x=a$, $y=a$ in the plane $z=0$

$\longleftrightarrow$