[MODEL QUESTION PAPER]

B. Tech I Year, I Sem,

MATHEMATICS - 1

Time - 3hr. Max. Marks - 100

Note:- The Question paper contains three sections, Section A, Section B & Section C with the weightage of 20, 30 & 50 marks respectively. Follow the instructions as given in each section.

SECTION - A

Note:- Attempt all questions (20×1)

Q.1. If $y = x^n$ then $y_n =$ ______

Q.2. If $y = xe^x$ then $y_n = xe^x + ne^x = e^x(x+n)$ (T/F)

Q.3. If $x = r\cos\theta$, $y = r\sin\theta$, ~~prove that~~ then $\frac{\partial r}{\partial x} =$ ______

Q.4. Euler's Theorem for homogenous fns. states that

______

Q.5. For any fn. $u = f_1(x,y)$ and $v = f_2(x,y)$, $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = 0$ (T/F)

Q.6. If u, v and w are independent then $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$

Q.7. When an error of +1% is made in measuring length and breadth of a rectangle then corresponding error in its area = ______%

Q.8. The condition for existence of point of minima ~~and or~~ maxima is $p =$ ______, $q =$ ______, $rt - s^2 =$ ______

Q.9. Any matrix A said to be orthogonal if ______

Q.10. The rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 5 & 7 \end{bmatrix}$ is ______

Q.11. A system of homogeneous linear equation ($AX=0$) is always consistent. (T/F)

Q.12. Cayley Hamilton theorem states that ______
______

Q.13. $\int_1^2 \left(\int_3^4 f(x,y)\,dx \right) dy = \int_3^4 \left(\int_1^2 f(x,y)\,dy \right) dx$ (T/F)

Q.14. The integral $\int_0^1 \int_0^{1-x} f(x,y)\,dy\,dx$ after changing the order of integration becomes ______

Q.15. Let the variables x, y in the double integral be changed to u, v by means of some relation then $dx\,dy =$ ______ $du\,dv$

Q.16. The value of $\Gamma \frac{1}{2} =$ ______

Q.17. Any vector $\vec{V}$ is said to be an ______ vector if curl $\vec{V} = 0$

Q.18. If ______, then vector $\vec{V}$ is called solenoidal vector point function.

Q.19. Gauss Divergence theorem give the relation between ______ integrals and ______ integral.

Q.20. Curl (grad ϕ) $= \nabla \times \nabla \phi =$ ______

SECTION-B

(3×10)

Note:- Attempt any three questions from this section.

Q.21. If $y = (\sin^{-1} x)^2$, prove that

$$(1-x^2)\,y_{n+2} - (2n+1)\,x\,y_{n+1} - n^2 y_n = 0$$

Also find $(y_n)_0$

Q.22. If $u = x(1-r^2)^{-1/2}$, $v = y(1-r^2)^{-1/2}$, $w = z(1-r^2)^{-1/2}$ where $r^2 = x^2+y^2+z^2$,

then show that $\dfrac{\partial(u,v,w)}{\partial(x,y,z)} = (1-r^2)^{-5/2}$

Q.23. Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$ and hence compute A^{-1}.

Also find the matrix represented by

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$

Q.24 Change into polar co-ordinates and evaluate

$$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dy\,dx.$$

Hence show that $\int_0^\infty e^{-x^2}\,dx = \dfrac{\sqrt{\pi}}{2}$

Q.25 Verify Stoke's theorem for the vector field $\vec{F} = (2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of $x^2+y^2+z^2=1$, bounded by its projection on the xy-plane.

SECTION-C

(2×5) each

Attempt any 2 parts from each question. All ques. are compulsory

Ques 26 → (a) If $y = a\cos(\log x) + b\sin(\log x)$, then show that:-

$$x^2 y_{n+2} + (2n+1)x y_{n+1} + (n^2+1) y_n = 0$$

(b) If $u = f(r)$ and $x = r\cos\theta$, $y = r\sin\theta$. Prove that:-

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

(c) Expand $\tan^{-1}\left(\frac{y}{x}\right)$ in the neighbourhood of (1,1) upto and inclusive of second degree terms. Hence compute $f(1.1, 0.9)$ approximately.

Ques 27: →

(a) If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$, Show that the Jacobian of y_1, y_2, y_3 w.r.t x_1, x_2, x_3 is 4.

(b) A balloon is in the form of right circular cylinder of radius 1.5 m and length 4.0 metre and is surmounted by hemispherical end. If the radius is increased by 0.01 m and length by 0.05 m, find the percentage change in the volume of balloon.

(c) ~~A Rectangular box, open at top, is to have a volume of 32 c.c. find the dimensions of the box requiring least material~~

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Ques 8: →

(a) Test for Consistency the following system of equations and if Consistent then solve them :—

$$x_1 + 2x_2 - x_3 = 3$$
$$3x_1 - x_2 + 2x_3 = 1$$
$$2x_1 - 2x_2 + 3x_3 = 2$$
$$x_1 - x_2 + x_3 = -1$$

(b) Use elementary transformation to reduce the following matrix A to triangular form and hence find the rank of A.

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

(c) If $A = \begin{bmatrix} 3 & 5+2i & -3 \\ 5-2i & 7 & 4i \\ -3 & -4i & 5 \end{bmatrix}$, show that A is Hermitian matrix and verify that iA is skew-Hermitian matrix.

Ques 19. →

Ans:

(a) Evaluate the integral $\int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$, changing the order of integration.

(b) Evaluate $\iint_R (x+y)^2 \, dx \, dy$, where R is the parallelogram in the x-y plane with the vertices $(1,0)$, $(3,1)$, $(2,2)$, $(0,1)$, using the transformation $u = x+y$, $V = x-2y$.

(c) State the Dirichlet's theorem for three variables. Hence evaluate the integral

$$\iiint x^{l-1} y^{m-1} z^{n-1} \, dx \, dy \, dz$$

where x, y, z are all positive but limited condition

$$\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1$$

Ques 30 :→

(a) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at point $(2,-1,2)$.

(b) Prove :-

$$\operatorname{div}(\operatorname{grad} r^n) = n(n+1)\, r^{n-2}$$

where $r = \sqrt{x^2+y^2+z^2}$

Hence show that $\nabla^2\left(\frac{1}{r}\right)=0$

(c) Find the directional derivative of $\phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point $P(1,1,1)$ in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$.