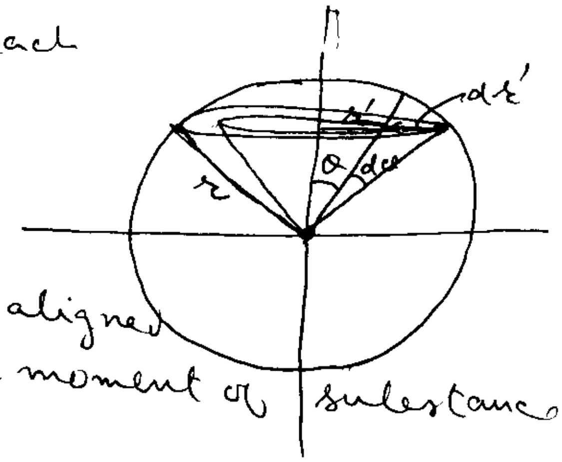
Langevin Theory of Paramagnetism

In para magnetic substances each atom/molecule is a magnetic dipole.



In the absence of external magnetic field all dipoles are aligned randomly so that net magnetic moment of substance is zero.

In presence of magnetising field, the dipoles align themselves at different angles to magnetic field $\vec{B}$

If μ_m = dipole moment of one atom

and $\vec{B}$ = Magnetic flux density

then a dipole at angle θ to $\vec{B}$ has Potential energy

$$U = -\mu_m B \cos\theta \quad \text{——— (1)}$$

All the dipoles within solid angle $d\Omega$ confined between angles θ & $\theta + d\theta$ will have same P.E.

Although $\vec{B}$ tends to align the dipole along B, the temperature of substance tends to decrease the alignment because of randomisation by temperature.

According to Maxwell Boltzman statistics the no of molecules (dipoles) having energy U at a temp. T and within solid angle $d\Omega$ per unit volume is given by

$$dn = C e^{-U/kT} \cdot d\Omega \qquad (C \text{ is constant}).$$

Now $d\Omega = \dfrac{\text{Area between } \theta \,\&\, \theta + d\theta}{r^2} = \dfrac{2\pi r' dr'}{r^2}$

$$= \frac{2\pi\, r\sin\theta \cdot r\,d\theta}{r^2}$$

$$d\Omega = 2\pi \sin\theta\, d\theta \quad \text{——— (2)}$$

$$\therefore dn = C e^{-\frac{U}{kT}} \cdot 2\pi \sin\theta\, d\theta$$

$$dn = C\, e^{\frac{u_m B \cos\theta}{kT}} \, 2\pi \sin\theta \, d\theta$$

$$= 2\pi C \, e^{\frac{u_m B \cos\theta}{kT}} \sin\theta \, d\theta$$

$$\underline{dn = A\, e^{x\cos\theta} \sin\theta \, d\theta} \quad \text{———} \quad (3)$$

where $A = 2\pi C$ ——(4)

and $x = \frac{u_m B}{kT}$ ——(5)

At a given temperature A & x are constants

③ $\Rightarrow \quad n = A \int_0^{\pi} e^{x\cos\theta} \sin\theta \, d\theta$.

put $t = \cos\theta$ so that when $\theta = 0$ $t = 1$ & when $\theta = \pi$ $t = -1$

$\Rightarrow \quad dt = -\sin\theta \, d\theta \Rightarrow \sin\theta \, d\theta = -dt$

$$\therefore n = -A \int_1^{-1} e^{xt} \, dt = -A \left[\frac{e^{xt}}{x} \right]_{+1}^{-1}$$

$$= -\frac{A}{x} \left[e^{-x} - e^{+x} \right]$$

$$= \frac{2A}{x} \left[\frac{e^{x} - e^{-x}}{2} \right]$$

$$\underline{n = \frac{2A}{x} \sinh x} \quad \text{———} \quad (6).$$

This is the number of molecules, all of which each of have orientation θ w.r.t B.

All these dn molecules in equation (3) have dipole moments in direction θ

Components of magnetic moment of each of these atoms along $\bar{B}'$ is $\mu_m \cos\theta$ where sine components cancel out each other.

$\therefore$ magnetic moment per unit volume along $\bar{B}'$

$$dM = \mu_m \cos\theta \, dn$$

$\therefore$ Magnetisation $M = \int_0^{\pi} dM.$

$$M = \mu_m \int_0^{\pi} A e^{x\cos\theta} \sin\theta \cdot \cos\theta \, d\theta.$$

Put $\cos\theta = u \Rightarrow -\sin\theta \, d\theta = du$

$$\Rightarrow M = -A\mu_m \int_{+1}^{-1} e^{xu} u \, du = -A\mu_m \int_{+1}^{-1} u e^{xu} du$$

$$= -A\mu_m \left[\frac{u e^{xu}}{x} - \int \frac{e^{xu}}{x} \right]_{+1}^{-1}$$

$$= -A\mu_m \left[-\frac{e^{-x}}{x} - \frac{e^{+x}}{x} \right] + \frac{A\mu_m}{x^2} \left[e^{xu} \right]_{+1}^{-1}$$

$$= +A\mu_m \left[\frac{e^{x} + e^{-x}}{x} \right] + \frac{A\mu_m}{x^2} \left[e^{-x} - e^{x} \right]$$

$$M = 2A\mu_m \underbrace{\left[\cosh x - \frac{1}{x} \sinh x \right]}_{} = \frac{2A\mu_m x}{3}$$ ← Langevin's function

Now $x = \frac{\mu_m B}{kT}$ $\quad x \propto \frac{B}{T}$.

$$\therefore M = \frac{2A}{3} \mu_m \frac{\mu_m B}{kT} \Rightarrow \boxed{M = \frac{2A}{3} \frac{\mu_m^2 B}{kT}}$$

$$M = \frac{2A\mu_m^2 \mu_0 H}{kT}$$

$$\boxed{\chi = \frac{M}{H} = \frac{2A\mu_m^2 \mu_0}{3kT}}$$