Transverse Nature of Electromagnetic waves

From Maxwells 3rd and 4th equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{--- (1)}$$

and $$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

In free space $\vec{J} = 0$.

$$\Rightarrow \nabla \times \vec{H} = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t} \quad \text{--- (2)}$$

Equations of Electric and magnetic fields are given by

$$E = E_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} \quad \text{---- (3)}$$

and $$H = H_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} \quad \text{---- (4)}$$

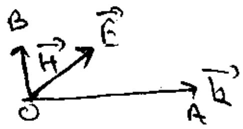
$$(3) \Rightarrow \nabla \times \vec{E} = i(\vec{k} \times \vec{E}) \quad \text{--- (5)}$$

$$(4) \Rightarrow \frac{\partial H}{\partial t} = -i\omega \vec{H} \quad \text{--- (6)}$$

$$(1), (5) \,\&\, (6) \Rightarrow i(\vec{k} \times \vec{E}) = i\mu\omega \vec{E}$$

$$\Rightarrow \vec{k} \times \vec{E} = \mu\omega \vec{H} \quad \text{--- (7)}$$

This shows that $\vec{H}$ is $\perp$ $\vec{k}$ as well as $\vec{E}$



$$\text{Also } (4) \Rightarrow \nabla \times \vec{H} = i(\vec{k} \times \vec{H}) \quad \text{--- (8)}$$

$$\& \ (3) \Rightarrow \frac{\partial E}{\partial t} = -i\omega \vec{E} \quad \text{--- (9)}$$

$$\therefore (2)\ (8) \,\&\, (9) \Rightarrow i(\vec{k} \times \vec{H}) = -i\varepsilon\omega \vec{E}.$$

$$\Rightarrow \vec{k} \times \vec{H} = -\varepsilon\omega \vec{E}.$$

$\therefore$ $\vec{E}$ is $\perp$ to $\vec{k}$ as well as $\vec{E}$

Since ~~both~~ $\vec{E}$ and $\vec{H}$ are $\perp$ to each other and both are $\perp$ to the propagation vector $\hat{k}$ i.e. direction of propagation of wave, therefore, e.m. waves are transverse in nature