Electromagnetic wave theory in free space :

Electromagnetic waves are coupled electric and magnetic oscillations at right angle to each other as well as at right angle to direction of propagation of waves.

They are associated with following properties

(a) E.m. waves travel with speed of light (3×10^8 m/s in vacuum)

(b) E.m. waves are transverse in nature.

(c) The ratio of electric to magnetic field ($\vec{E}/\vec{B}$) in e.m. wave equals velocity of light.

(d) E.m. wave carry both, energy and momentum which can be delivered to a given space.

Wave Equation in free space

Free space is a perfect dielectric containing no charge ($\rho=0$) and no conduction current ($\vec{J}=0$).

For this case

$$\nabla\cdot\vec{D} = 0 \Rightarrow \nabla\cdot\vec{E} = 0 \qquad \text{---(1)}$$

$$\nabla\cdot\vec{B} = 0 \qquad \text{---(2)}$$

$$\nabla\times\vec{E} = -\frac{\partial B}{\partial t} \qquad \text{---(3)}$$

$$\nabla\times\vec{H} = \frac{\partial D}{\partial t} \qquad \text{---(4)}$$

For free space $\mu=\mu_0$, $\varepsilon=\varepsilon_0$ $\sigma=0$, $\rho=0$.

Also
$$\left.\begin{aligned}\vec{D} &= \varepsilon_0\vec{E}\\ \& \quad \vec{B} &= \mu_0\vec{H}\end{aligned}\right\} \text{---- (5)}$$

Taking curl of equation (3)

$$\nabla\times(\nabla\times\vec{E}) = -\frac{\partial}{\partial t}(\nabla\times\vec{B})$$

$$= -\mu_0\frac{\partial}{\partial t}(\nabla\times\vec{H})$$

$$= -\mu_0\frac{\partial}{\partial t}\left(\frac{\partial\vec{D}}{\partial t}\right)$$

$$\nabla\times(\nabla\times\vec{E}) = -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \qquad (\vec{D}=\varepsilon_0\vec{E}).$$

$$\Rightarrow \underbrace{\nabla(\nabla\cdot\vec{E})}_{0} - \nabla^2\vec{E} = -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\Rightarrow -\nabla^2\vec{E} = -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}.$$

$$\Rightarrow \quad \nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \text{——— (6)}$$

Similarly taking curl of equation (4) we get

$$\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} \qquad \text{——— (7)}$$

Equations (6) and (7) are wave equations for electric and magnetic fields.

** Expansion of eqn (6) in cartesian form

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_z}{\partial t^2}$$

Similarly eqn (7) can be expanded

In general e.m. wave consist of electric and magnetic fields which are ⊥ to each other as well as direction of propagation of wave.

These waves are called <u>uniform plane waves</u>.

Propagation of e m waves through free space
&
Expression for velocity of electromagnetic waves and refractive index:

For plane e.m. waves

$$\nabla \cdot \vec{D} = 0 \qquad (1)$$

$$\nabla \cdot \vec{B} = 0 \qquad (2)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (3)$$

and $$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \qquad (4)$$ $\left[\because \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}\right]$ $\vec{J} = 0$ for free space

For free space $\mu = \mu_0 \quad \varepsilon = \varepsilon_0 \; , \sigma = 0 , \; \rho = 0$

$$D = \varepsilon_0 E \quad \text{and} \quad B = \mu_0 H .$$

Taking curl of eqn (3)

$$\nabla \times \nabla \times \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

$$= -\mu_0 \frac{\partial}{\partial t}(\nabla \times \vec{H})$$

$$= -\mu_0 \frac{\partial}{\partial t}\left(\frac{\partial \vec{D}}{\partial t}\right)$$

$$\nabla \times (\nabla \times \vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla(\nabla \cdot E) - \nabla^2 E = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

But $\nabla \cdot E = \frac{1}{\varepsilon_0} \nabla \cdot D = 0$

$$\Rightarrow \quad -\nabla^2 E = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \qquad (5)$$

Similarly

$$\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} \qquad (6)$$

Equation (5) and (6) are vector equations of same nature and hence $\vec{E}$ and $\vec{H}$ satisfy scaler wave equation of the type

$$\nabla^2 u = \mu_0 \varepsilon_0 \frac{\partial^2 u}{\partial t^2}$$ where u is scalar function

General wave equation is given by

$$\nabla^2 u = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$$

$\Rightarrow \quad c^2 = \frac{1}{\mu_0 \varepsilon_0} \quad \Rightarrow c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ Also $v = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \mu_r \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$

$\sqrt{\mu_r \varepsilon_r} = \frac{c}{v}$ = a refractive index

$\Rightarrow$ Ref. index = $\sqrt{\mu_r \varepsilon_r}$