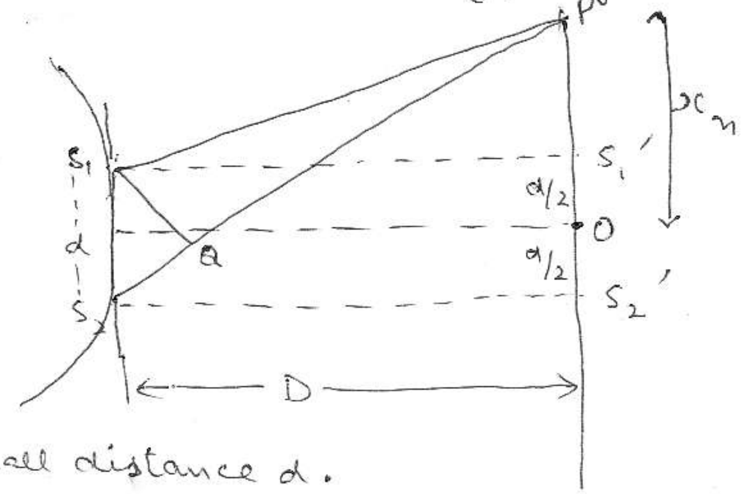
Young's Double Slit Experiment and expression of fringe width



In double slit experiment two points of same wave front act as coherent

S

sources

A monochromatic source of light S illuminates two slits S_1 & S_2 seperated by small distance d.
Let waves from S_1 and S_2 meet at point P on a Screen at distance D from sources.

Let O be a point symmetric w.r.t S_1 and S_2 so that waves from S_1 & S_2 always meet at O in same phase and produce bright fringe.

Let x_n be the distance of nth fringe from O at pt P,

Path difference $S_2Q = S_2P - S_1P$ ——— ①

From figure $S_2'P = x_n + d/2$ & $S_1'P = x_n - d/2$ ——— ②

Now $S_2P = \left(S_2S_2'^2 + S_2'P^2\right)^{1/2} = \left[D^2 + (x_n + d/2)^2\right]^{1/2}$

$$\Rightarrow S_2P = D\left[1 + \frac{(x_n + d/2)^2}{D^2}\right]^{1/2}$$

$$= D\left[1 + \frac{(x_n + d/2)^2}{2D^2}\right] \quad \text{(Using Binomial Theorem)}$$

$$S_2P = D + \frac{(x_n + d/2)^2}{2D} \quad \text{——— (3)}$$

Similarly

$$S_1P = D + \frac{(x_n - d/2)^2}{2D} \quad \text{——— (4)}$$

$$\therefore \text{Path diff} = S_2P - S_1P = \frac{(x_n + d/2)^2 - (x_n - d/2)^2}{2D}$$

$$= \frac{4x_n d/2}{2D} = \frac{2x_n d}{2D}.$$

$$\Rightarrow \text{Path difference} = \frac{x_n d}{D} \quad \text{——— (5)}$$

For Bright fringes

Path diff = $2n\lambda/2$ —— (6)

(5) & (6) $\Rightarrow$ $\frac{x_n d}{D} = n\lambda$

$$x_n = \frac{n\lambda D}{d} \quad \text{—— (7)}$$

gives position of nth bright fringe

$\therefore$ Position of $(n-1)$th bright fringe is given by

$$x_{n-1} = \frac{(n-1)\lambda D}{d} \quad \text{—— (8)}$$

Difference between positions of two successive bright fringes gives fringe width β of dark fringe

$$\therefore \beta = x_n - x_{n-1} = \boxed{\frac{\lambda D}{d}} \quad \text{—— (9)}$$

For Dark fringes

Path difference $= (2n-1)\lambda/2$ —— (10)

(5) & (10) imply that

$$\frac{x_n d}{D} = (2n-1)\lambda/2$$

$$x_n = \frac{(2n-1)\lambda D}{2d}$$ gives position of nth dark fringe

$$\Rightarrow x_{n-1} = \frac{(2n-3)\lambda D}{2d}$$

$\Rightarrow$ fringe width of bright fringes is

$$\beta' = x_n - x_{n-1}$$

$$\boxed{\beta' = \frac{\lambda D}{d}}$$

This shows that bright and dark fringes have equal width.

$$\beta = \beta' = \frac{\lambda D}{d}$$

$$\beta \propto \lambda$$
$$\beta \propto D$$
$$\beta \propto \frac{1}{d}$$

Note:- Larger the β & β', easier it is to see fringes. Therefore, for fringes to be distinctly seen D should be large & d should be small

For small value of D & large value of d, β will be very small and it will be difficult to resolve fringes.