Momentum and Energy Relationship (Relativistic)

$$E^2 - p^2c^2 = (mc^2)^2 - c^2(mv)^2 \qquad (E = mc^2)$$

$$= \left(\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2 - c^2\left(\frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2$$

$$= \frac{m_0^2c^4 - m_0^2v^2c^2}{\left(1-\frac{v^2}{c^2}\right)} = \frac{m_0^2c^2(c^2-v^2)c^2}{(c^2-v^2)}$$

$$E^2 - p^2c^2 = m_0^2c^4 \quad \text{--- (1)}$$

$$E^2 = p^2c^2 + m_0^2c^4 \quad \text{--- (2)}$$

Also $E = K + m_0c^2$ (m_0c^2 = Rest mass Energy, K = K.E)

$$\therefore (K + m_0c^2)^2 = p^2c^2 + m_0^2c^4 \quad \text{--- (3)}$$

- Differentiating (1) w.r.t p

$$2E\frac{dE}{dp} = 2pc^2 + 0$$

$$\Rightarrow E\frac{dE}{dp} = pc^2$$

$$mc^2\frac{dE}{dp} = m\cancel{v}c^2$$

$$\Rightarrow \frac{dE}{dp} = v$$

Rate of change of energy with momentum is equal to the speed of the body