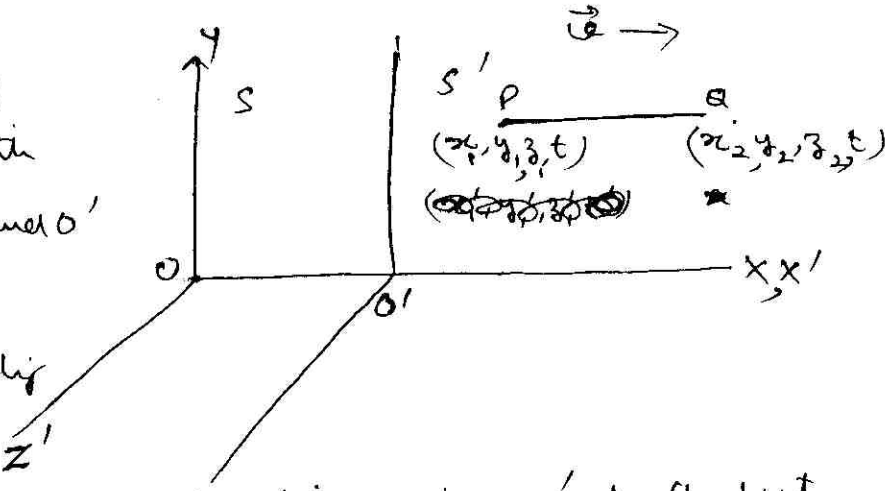
Length contraction

Let S and S' be inertial frames of references with observers at orgins O and O' respectively

S' moves with velocity $\vec{v}$ along x-axis



Let a rod PQ ∥ to x-axis be fixed in S' so that it is at rest in S' and moves in S with velocity $\vec{v}$

$\Rightarrow x_2' - x_1' = l_0$ $\quad x_2'$ & x_1' are co-ordinates of Q & P in S'

$\Rightarrow x_2 - x_1 = l$ $\quad x_2$ & x_1 " " " Q & P ∥ S

From Lorentz transformation equations at time t

$$x_2' = \frac{x_2 - vt}{\sqrt{1 - v^2/c^2}}$$

and $$x_1' = \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow x_2' - x_1' = \frac{(x_2 - x_1)}{\sqrt{1 - v^2/c^2}}$$

$$l_0 = \frac{l}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow l = l_0 \sqrt{1 - v^2/c^2}$$