Mass Energy relation (Equivalence of mass and energy)

Let m_0 = rest ~~mass~~ mass of particle.

m = mass of moving particle

Let u = velocity of particle at any instant under ~~sotted~~ the action of constant force such that

$$F = \frac{d}{dt}(mu) = m\frac{du}{dt} + u\frac{dm}{dt} \qquad (\because m \,\&\, u \text{ both are variable})$$

Gain in ~~kinetic~~ energy when force F acts through distance dx is

$$dK = F\,dx = m\frac{du}{dt}.dx + u\frac{dm}{dt}.dx$$

$$\Rightarrow dK = mu\,du + u^2 dm \quad \text{———} \quad (1)$$

Also $m = \dfrac{m_0}{\sqrt{1-u^2/c^2}}$

$$\Rightarrow m^2\left(1-\frac{u^2}{c^2}\right) = m_0^2$$

$$m^2(c^2-u^2) = m_0^2c^2 \Rightarrow m^2c^2 - m^2u^2 = m_0^2c^2$$

Differentiating

$$c^2\,2m\,dm - m^2.2u\,du - u^2\,2m\,dm = 0$$

$$c^2 dm = mu\,du + u^2 dm \quad \text{———} \quad (2)$$

$(1)\,\&\,(2) \Rightarrow dK = c^2 dm$

integrating both sides

$$\int_0^K dK = c^2\int_{m_0}^{m} dm$$

$$K = c^2(m-m_0)$$

Gain in K.E = $(m-m_0)c^2$

m_0c^2 represents rest mass energy

$(m-m_0)$ = change in mass which is as a result of change in energy

$\Rightarrow$ mc^2 must represent the energy possessed by body when it is moving

$\Rightarrow$ $\boxed{E = mc^2}$ or $\underline{\Delta E = \Delta m c^2}$