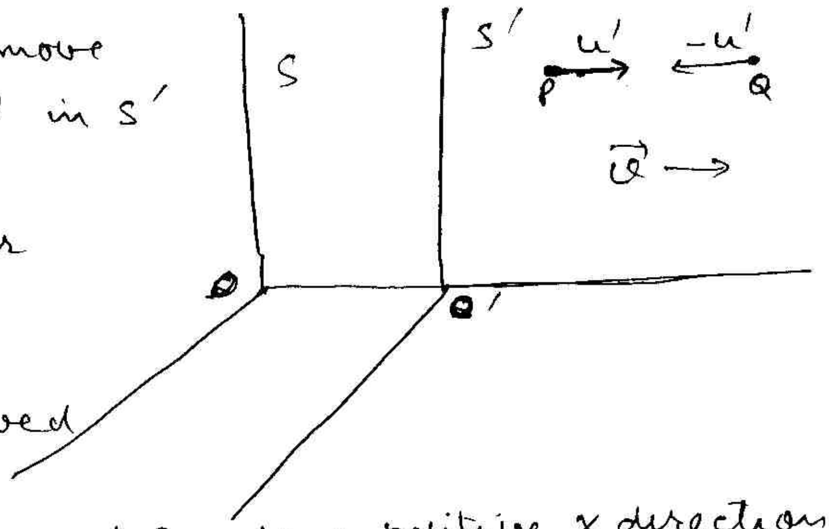
Variation of mass with velocity or Relativistic mass

Let two identical balls move with velocities u' & $-u'$ in S' || to x-axis.

Let u_1 and u_2 be their velocities w.r.t S and m_1 and m_2 be their masses as observed in S



Let v = velocity of S' w.r.t S along positive x direction.

After colliding the two particles will come to rest w.r.t S' and will move with velocity v w.r.t S.

Applying law of conservation of momentum in S

$$m_1u_1 + m_2u_2 = (m_1+m_2)v$$

$$\Rightarrow \quad m_1(u_1-v) = m_2(v-u_2) \quad \text{———} (1).$$

Now $$u_1 = \frac{u'+v}{1+\frac{u'v}{c^2}} \quad \text{———} (2)$$

& $$u_2 = \frac{-u'+v}{1-\frac{u'v}{c^2}} \quad \text{———} (3)$$

$$\Rightarrow \quad m_1\left(\frac{u'+v}{1+\frac{u'v}{c^2}} - v\right) = m_2\left(v - \frac{v-u'}{1-\frac{u'v}{c^2}}\right)$$

$$\Rightarrow \quad m_1\left(\frac{u' - \frac{u'v^2}{c^2}}{1+\frac{u'v}{c^2}}\right) = m_2\left(\frac{-\frac{u'v^2}{c^2}+u'}{1-\frac{u'v}{c^2}}\right)$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{1+\frac{u'v}{c^2}}{1-\frac{u'v}{c^2}} \quad \text{———} (4)$$

(2) $\Rightarrow$ $$1-\frac{u_1^2}{c^2} = 1 - \frac{1}{c^2}\left(\frac{u'^2+v^2+2u'v}{1+\frac{u'^2v^2}{c^4}+\frac{2u'v}{c^2}}\right)$$

$$1-\frac{u_1^2}{c^2} = \frac{c^2 + \frac{u'^2v^2}{c^2} + \cancel{2u'v} - u'^2 - v^2 - \cancel{2u'v}}{c^2\left(1+\frac{u'v}{c^2}\right)^2} \quad \text{—} (5)$$

Similarly

$$1 - \frac{u_2^2}{c^2} = \frac{c^2 + \frac{u'^2 u^2}{c^2} - u'^2 - u^2}{c^2\left(1 + \frac{u' u}{c^2}\right)^2}$$

$$\Rightarrow \frac{1 - \frac{u_2^2}{c^2}}{1 - \frac{u_1^2}{c^2}} = \frac{\left(1 + \frac{u' u}{c^2}\right)^2}{\left(1 - \frac{u' u}{c^2}\right)^2}$$

$$\Rightarrow \frac{1 + \frac{u' u}{c^2}}{1 - \frac{u' u}{c^2}} = \frac{\sqrt{1 - \frac{u_2^2}{c^2}}}{\sqrt{1 - \frac{u_1^2}{c^2}}} \quad \text{--- (6)}$$

(4) & (5) $\Rightarrow$ $$\frac{m_1}{m_2} = \frac{\sqrt{1 - u_2^2/c^2}}{\sqrt{1 - u_1^2/c^2}}$$

If velocities are so chosen that that $u_2 = 0$, then for 2nd particle, $m_2 = m_0$

$$\therefore \frac{m_1}{m_0} = \frac{1}{\sqrt{1 - u_1^2/c^2}} \quad \text{--- (7)}$$

Since both particles are considered to be identical, therefore, rest mass of particle 1 will also be m_0. Therefore equation (7) is valid for single particle also

$\Rightarrow$ if m = mass of particle ~~moving~~ & it moves with velocity u then

$$\frac{m}{m_0} = \frac{1}{\sqrt{1 - u^2/c^2}}$$

$$\Rightarrow m = \frac{m_0}{\sqrt{1 - u^2/c^2}} \quad \Rightarrow m > m_0$$

<u>Special cases</u>

i) $u <<< c \Rightarrow m \simeq m_0$

ii) $u = c \Rightarrow m = \infty$

iii) $u > c \Rightarrow m$ is imaginary.

$\Rightarrow$ No physical particle can have velocity $\geq c$.