

Addition of velocities:

Let a frame S' move with uniform velocity u relative to S along $+x$ direction.

Let u and u' be the velocities of a particle as observed by observers in S & S' .

Inverse transformation equations are given by

$$x = \frac{x' + ut'}{\sqrt{1 - \frac{u^2}{c^2}}} \quad y = y' \quad z = z'$$

$$\text{and } t = \frac{t' + \frac{ux'}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$dx = \frac{dx' + u dt'}{\sqrt{1 - \frac{u^2}{c^2}}} \quad dy = dy' \quad dz = dz'$$

$$dt = \frac{dt' + \frac{u dx'}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$\Rightarrow \frac{dx}{dt} = \frac{dx' + u dt'}{dt' + \frac{u dx'}{c^2}} = \frac{\frac{dx'}{dt'} + u}{1 + \frac{u}{c^2} \frac{dx'}{dt'}}$$

$$\Rightarrow u_x = \frac{u_x' + u}{1 + \frac{u}{c^2} u_x'} \quad \text{--- (1)}$$

Similarly

$$u_y = \frac{u_y'}{\beta \left(1 + \frac{u}{c^2} u_x'\right)} \quad \& \quad u_z = \frac{u_z'}{\beta \left(1 + \frac{u}{c^2} u_x'\right)}$$

$$\text{where } \beta = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

If object moves with velocity u' along x axis then $u_x = u$, $u_y = 0$, $u_z = 0$

$$u = \frac{u' + u}{1 + \frac{u u'}{c^2}}$$

$$\text{Also } u' = \frac{u - u}{1 - \frac{u u}{c^2}} \quad (-: u \rightarrow -u)$$