

BCS Theory: - It is the theory of superconductivity

~~was~~ developed by Bardeen, Cooper & Schrieffer\*

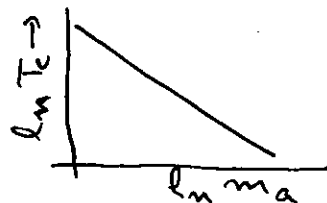
It is based on two experimental results

ie i) Isotopic effect

ii) Variation of electronic specific heat with temperature.

According to Isotopic effect, it was found that with critical temperature  $T_c$  and isotopic mass  $m_a$  are related by

$$m_a^\alpha T_c = \text{constant}$$



where  $\alpha$  is a material specific constant and is called isotopic effect co-efficient and is given by

$$\alpha = \frac{\partial \ln T_c}{\partial \ln m_a} = \frac{\partial y}{\partial x}$$

where  $y = \ln T_c$

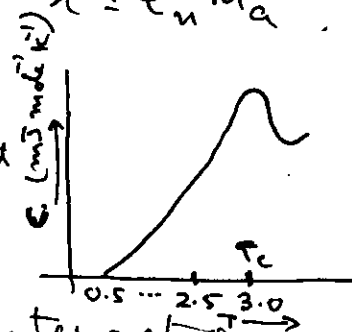
$x = \ln m_a$

Value of  $\alpha$  varies from 0.45 to 0.50

Also in superconductors specific heat

varies exponentially & is given by

$$C_v = e^{-AT/T_c} \text{ where } A \text{ is const.}$$



According to BCS Theory, there is an interaction of two electrons through phonons\*\*

when an electron approaches an ion in the lattice, there is coulomb attraction between electron and the lattice ion.

This attraction between electron and lattice ion produces lattice distortion which causes an increase in the density of ions in the region

\* Bardeen  $\rightarrow$  John Bardeen

Cooper  $\rightarrow$  Leon N Cooper

Schrieffer  $\rightarrow$  John R Schrieffer.

\*\* Phonon: It is a quanta of lattice vibration

of distortion.

The higher density of ions in distorted region attracts another electron.

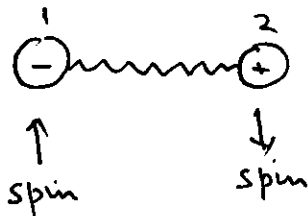
Hence a free electron exerts a small attractive force on another electron through phonons.

A pair of electrons are, therefore, coupled through a phonons.

This pair of free electrons coupled through phonons and having small attractive force ~~is~~ called Cooper pair.

Energy of Cooper pair is less than the energy of two individual electrons. These electrons of Cooper pair have opposite spins making its total spin zero.

Hence the electrons in superconductor are bosons



when there is no current in superconductor, the linear momentum of electrons in Cooper pair are equal and opposite making total momentum zero.

Energy gap  $E_g$  of Cooper pair is  $\approx 10^{-3}$  eV at 0 (K).

Also energy gap  $E_g$  of superconductor is given by

$$E_g(0) = 3.53 k_B T_c$$

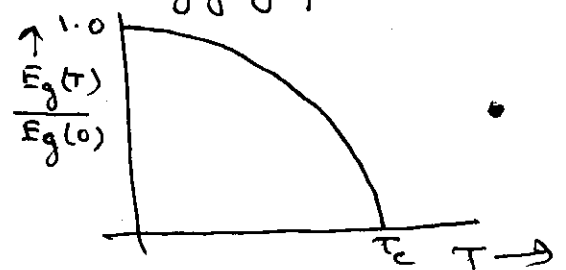
where  $k_B$  is Boltzmann constt &  $T_c$  = critical Temp.

At  $T > 0$  (K) some Cooper pairs break up.

These individual electrons interact with remaining Cooper pairs and reduce the energy gap as shown in fig.

$$\text{At } T = T_c \quad E_g(T) = 0$$

and there are no more Cooper pairs, so that material loses superconducting property



Critical Temperature of simple ~~and~~ conventional superconductors is given by BCS formula

$$k_B T_c = 1.13 \hbar \omega_D e^{-1/\lambda_{ep}}$$

where  $\omega_D$  is Debye or characteristic frequency which varies from metal to metal

and  $\lambda_{ep}$  is dimensionless electron phonon coupling parameter. (its order is  $\sim 0.3$ )

Note:  $\omega_D$  is related to Debye temperature  $\Theta_D$  by

$$k_B \Theta_D = \hbar \omega_D$$

$\Theta_D$  ranges from 100 K to 500 K.