Derivation of Meissner effect from 2nd London Equation

From 2nd London equation

$$\nabla \times \vec{J_s} = -n_s \frac{e^2}{m} \vec{B} \quad \text{---(1)}$$

and from Maxwell equation

$$\nabla \times \vec{B} = \mu_0\left(J_s + \frac{\partial D}{\partial t}\right)$$

But for superconductors $\frac{\partial D}{\partial t} = 0 \quad (\because D = 0)$.

$$\Rightarrow \nabla \times \vec{B} = \mu_0 J_s.$$

Taking curl of both sides

$$\nabla \times \nabla \times \vec{B} = \mu_0 \nabla \times \vec{J_s}.$$

$$-\nabla^2 B = \mu_0 \left(-\frac{n_s e^2}{m}\right)\vec{B}$$

$$\Rightarrow \nabla^2 \vec{B} = \frac{\mu_0 n_s e^2}{m} . \vec{B} \quad \text{---(2)}$$

$$\nabla^2 \vec{B} = \frac{1}{\lambda^2} \vec{B} \; * \quad \text{----(3)}$$

London

where λ is called London Penetration depth.

$$\& \quad \lambda = \left(\frac{m}{\mu_0 n_s e^2}\right)^{1/2} \quad \text{———(4)}$$

(4) $\Rightarrow$ that magnetic field does not abruptly falls to zero at the surface of superconductor but penetrates with exponential attenuation into the material.

Also $\lambda \propto \frac{1}{\sqrt{n_s}}$

---

* In equation (3) if $B = B_0 e^{-x/\lambda}$ i.e. B is function of x

$$\Rightarrow \frac{\partial B}{\partial x} = -\frac{1}{\lambda} B_0 e^{-x/\lambda} \quad \& \quad \frac{\partial^2 B}{\partial x^2} = \frac{1}{\lambda^2} B_0 e^{-x/\lambda}$$

$$\Rightarrow \frac{\partial^2 B}{\partial x^2} = \frac{1}{\lambda^2} B \quad \Rightarrow B = B_0 e^{-x/\lambda}$$ satisfies (3) & hence is solution of equation (3)