De Broglie concept of matter waves

According to De Broglie, matter has dual nature ie it behaves as particle as well as waves. According to him

i) Matter and light, both are form of energy and each of them can be transformed into each other.

ii) Both are governed by the space time symmetries of the theory of relativity.

A moving matter particle is surrounded by a wave whose wavelength λ depends on the mass of the particle and its velocity.

The waves associated with matter particles are known as matter waves or De-Broglie waves.

The wavelength λ and momentum p of particle are related to each other by

$$\lambda = \frac{h}{p}$$

Note:

For particle $\lambda = \frac{h}{mv}$

For e.m. wave $\lambda = \frac{h}{mc}$

Wavelength associated with particle accelarated through Potential V

We know that $E = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$

$$\Rightarrow p = \sqrt{2mE}$$

$$\therefore \lambda = \frac{h}{\sqrt{2mE}} \quad \text{——— (1)} \qquad (\because \lambda = h/p)$$

If a particle of charge q moves under P.D V then

$$E = qV$$

$$\therefore \lambda = \frac{h}{\sqrt{2mqV}} \quad \text{——— (2)}$$

For electron $q = e \Rightarrow \lambda = \frac{h}{\sqrt{2meV}}$

Also for a gas at temp T in thermal equilibrium

$$E = \frac{1}{2}mv^2 = \frac{3}{2}KT \qquad (K \text{ is Boltzman const})$$

$$\therefore \lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{3mKT}}$$

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Velocity of de-Broglie waves :—

According to Debroglie, a wave of wavelength λ is associated with a particle moving with a velocity v such that

$$\lambda = \frac{h}{mv}$$

Also $E = h\nu = mc^2$

$$\Rightarrow \nu = mc^2/h$$

Now velocity of ~~wave~~ Debroglie wave

$$v_p = \nu\lambda = \frac{mc^2}{h} \cdot \frac{h}{mv}$$

$$v_p = \frac{c^2}{v}$$

From theory of relativity $v < c$

$$\therefore v_p > c$$

This seems to be against Theory of relativity according to which nothing can have velocity larger than velocity of light.

Concept of phase velocity and group velocity

Expression $v_p = c^2/v$ brings us in conflict with theory of relativity. In fact velocity of Debroglie waves may be considered as the velocity of group of waves of slightly different angular frequencies and slightly different wave numbers superimposing over each other to give constructive interference in a small region within which the particle may be localised.

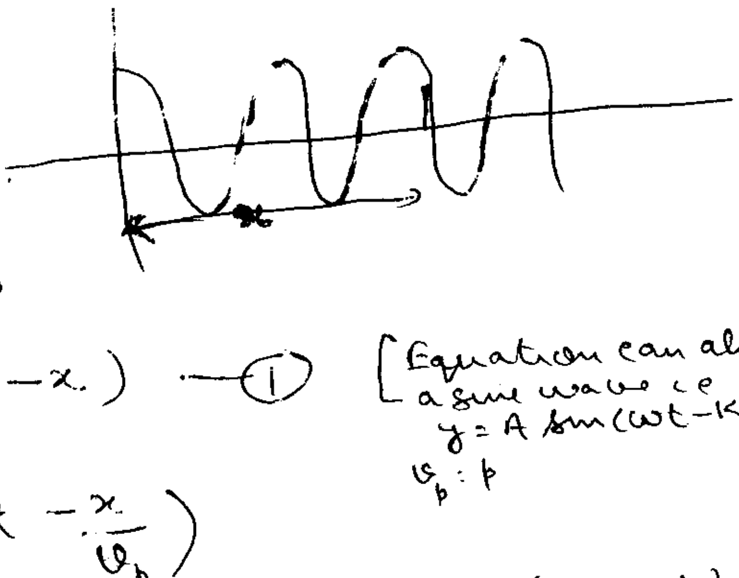
Outside this region the waves interfere destructively. Velocity of this group of waves is called <u>group velocity</u>.

This region of constructive interference of group of waves moves with velocity of particles. Hence group velocity is equal to the velocity of particle.

Velocity of individual waves is called <u>phase velocity</u>, given by c^2/v and has no physical significance.

General equation of waves

Waves are the result of simple harmonic vibration in a medium. The general equation of wave is given by



$$y = A \cos \frac{2\pi}{\lambda} (v_p t - x) \quad \text{— (1)}$$

[Equation can also be a sine wave i.e. $y = A \sin(\omega t - Kx)$
$v_p : p$

$$y = A \cos \frac{2\pi}{\lambda} v_p \left(t - \frac{x}{v_p} \right)$$

$(v_p = \nu \lambda)$

$$y = A \cos 2\pi \nu \left(t - \frac{x}{v_p} \right) \quad \text{— (2)}$$

$$y = A \cos \left(2\pi \nu t - \frac{2\pi \nu x}{\nu \lambda} \right)$$

$$y = A \cos \left(\omega t - \frac{2\pi}{\lambda} . x \right)$$

$$\Rightarrow y = A\cos(\omega t - kx) \quad \text{——} (3)$$

where $\omega = 2\pi\nu$ is called angular frequency and $k = \frac{2\pi}{\lambda}$ is called wave number.