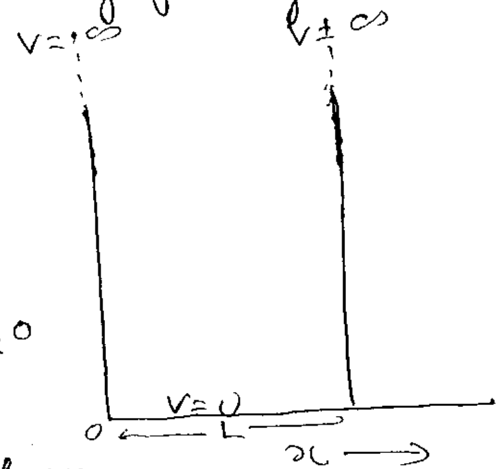
Particle in a box (Infinite Square well Potential):-

Consider a particle moving freely inside a box along x-axis



L = length of the box

for $x > 0$ and $x < L$ $V = 0$

$\Rightarrow V = 0$ for $0 < x < L$

and $V = \infty$ for $x \geq L$ and $x \leq 0$

This means that particle cannot exist outside the box

∴ outside the box $\psi = 0$

and inside the box

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E \psi = 0 \quad \text{------ (1)}$$

$$\frac{\partial^2 \psi}{\partial x^2} + K^2 \psi = 0 \text{ -- (2) where } K^2 = \frac{2mE}{\hbar^2} \Rightarrow K = \frac{\sqrt{2mE}}{\hbar} \quad \text{— (3)}$$

[redacted]

General solution of this eqn is

$$\psi = A \sin Kx + B \cos Kx \quad \text{———— (4)}$$

using boundary conditions

$\psi = 0$ at $x = 0$ $\Rightarrow$ $0 = B$.

$$\Rightarrow \psi = A \sin Kx \quad \text{------------ (5)}$$

Also $\psi = 0$ at $x = L$

$\Rightarrow$ $0 = A \sin KL$

Since ψ is finite $A \neq 0$

∴ $\sin KL = 0 = \sin n\pi$ $\quad n = 1, 2,$

$$KL = n\pi$$

$$K = \frac{n\pi}{L} \quad \text{---- (6)}$$

(3) & (6) $\Rightarrow$ $\dfrac{n^2\pi^2}{L^2} = \dfrac{2mE_n}{\hbar^2}$

$$E_n = \frac{n^2 \hbar^2 \pi^2}{2mL^2} = \frac{n^2 h^2}{8mL^2}$$

$$E_n = \frac{n^2 h^2}{8mL^2} \quad \text{----- (7)} \qquad n = 1, 2, 3 \ldots\ldots$$

- This shows that particle inside a box can have only discrete energies $\frac{h^2}{8mL^2}, \frac{4h^2}{8mL^2}, \frac{9h^2}{8mL^2} \ldots\ldots$

Each permitted energy is called energy eigen value of the particle and constitutes energy level. Corresponding function is called energy eigen function

$$\psi_n = A \sin \frac{n\pi}{L} x \qquad \left(\because k = \frac{n\pi}{L} \right)$$

To find A, let us apply normalisation condition

$$\int_{-\infty}^{+\infty} |\psi_n|^2 dx = 1 \Rightarrow \int_0^L |\psi_n|^2 dx = 1$$

$$\Rightarrow A^2 \int_0^L \sin^2 kx \, dx = 1$$

$$\Rightarrow A^2 \int_0^L \left(\frac{1 - \cos 2kx}{2} \right) dx = 1.$$

$$\Rightarrow \frac{A^2}{2} \int_0^L 1 \, dx - \frac{A^2}{2} \int_0^L \cos 2kx = 1$$

$$\Rightarrow \frac{A^2 L}{2} - \frac{A^2}{2} \left| \frac{\sin 2 \frac{n\pi}{L} x}{2k} \right|_0^L = 1 \Rightarrow \frac{A^2 L}{2} - 0 = 1$$

$$\Rightarrow \frac{A^2 L}{2} = 1$$

$$\Rightarrow A = \sqrt{\frac{2}{L}}$$

$$\therefore \quad \psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x$$

Although ψ_n may be +ve or -ve $|\psi_n|^2$ is always +ve

Also since ψ_n is normalised, its value at a given x gives the probability density of finding the particle there.