- **Schrödinger Wave Equation**: It is a fundamental equation of wave mechanics in the same manner as Newton's laws are fundamental to classical mechanics.

Schrodinger equation is in fact differential form of de Broglie wave equation.

wave function of a particle is given by

$$\Psi = A e^{-\frac{i}{\hbar}(Et - px)} \quad \text{------- (1)}$$

On differentiating partially w.r.t. t we get

$$E\Psi = i\hbar \frac{\partial \Psi}{\partial t} \quad \text{---- (2)}$$

On partially differentiating (1) w.r.t. x we get

$$p\Psi = \frac{\hbar}{i} \frac{\partial \Psi}{\partial x} \quad \text{------ (3)}$$

Now total energy E of a particle is the sum of K.E E_k and Potential energy V.

$$E = E_k + V \quad \text{------ (4)}$$

$$\Rightarrow E\Psi = E_k\Psi + V\Psi \quad \text{------ (5)}$$

$$\text{Now } E_k = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2$$

$$\Rightarrow i\hbar \frac{\partial \Psi}{\partial t} = \frac{1}{2m}\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2 \Psi + V\Psi$$

$$\Rightarrow i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi \quad \text{—— (6)}$$

This is Schrödinger's time dependent equation in one dimention.

$$\text{Now } \Psi = A e^{-i/\hbar\,(Et - px)} = A e^{-\frac{i}{\hbar}Et} e^{\frac{ipx}{\hbar}}$$

At $t = 0$ $\Psi = \Psi_0$

$$\Psi_0 = A e^{i/\hbar\, px}$$

$$\Rightarrow \Psi = \Psi_0 e^{-\frac{iEt}{\hbar}} \quad \text{—— (7)}$$

Differentiating eqn (7) w.r.t t partially

$$\frac{\partial \Psi}{\partial t} = -i\Psi_0 \frac{E}{\hbar} e^{-\frac{iEt}{\hbar}}$$

$$\frac{\partial \Psi}{\partial t} = -\frac{iE}{\hbar} \Psi_0 e^{-\frac{iEt}{\hbar}} \quad \text{——— (8)}$$

Differentiating eqn (7) w.r.t x partially

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{\partial^2 \Psi_0}{\partial x^2} \cdot e^{-\frac{iEt}{\hbar}} \quad \text{- - - - (9)}$$

(6), (7), (8) and (9)

$$\Rightarrow i\hbar\left(-\frac{iE}{\hbar}\right)\Psi_0 e^{-\frac{iEt}{\hbar}} = -\frac{\hbar^2}{2m} \cdot \frac{\partial^2 \Psi_0}{\partial x^2} e^{-\frac{iEt}{\hbar}} + V\Psi_0 e^{-\frac{iEt}{\hbar}}$$

$$\Rightarrow E\Psi_0 = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi_0}{\partial x^2} + V\Psi_0$$

$$\Rightarrow \frac{\hbar^2}{2m}\frac{\partial^2 \Psi_0}{\partial x^2} + (E-V)\Psi_0 = 0 \Rightarrow \frac{\partial^2 \Psi_0}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\Psi_0 = 0$$

This is Time Independent Shrodinger's wave equation

Note: 1. In 3-dimension

$$\nabla^2 \Psi_0 + \frac{2m}{\hbar^2}(E-V)\Psi_0 = 0$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Note 2 For a free particle $V = 0$

$$\Rightarrow \nabla^2 \Psi_0 + \frac{2m}{\hbar^2} E\Psi_0 = 0$$