UNIT-I LONG ANSWER TYPE QUESTIONS :→

Ques 1 :→ If $y = \sin(m \sin^{-1} x)$ then prove that

$$(1-x^2)y_2 - xy_1 + m^2 y = 0$$

and $$(1-x^2)y_{n+2} = (2n+1)x y_{n+1} + (n^2 - m^2) y_n$$

Ques 2 :→ If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$ then Prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$$

Ques 3 :→ If $u = \sin^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$ then prove that

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$

Ques 4 :→ Transform the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar-coordinates.

Ques 5 :→ If $x+y = 2e^{\theta}\cos\phi$ and $x-y = 2ie^{\theta}\sin\phi$ then prove that $\frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^2 v}{\partial \phi^2} = 4xy\frac{\partial^2 v}{\partial x \partial y}$.

Ques 6 :→ Expand $\tan^{-1}\frac{y}{x}$ in the neighbourhood of $(1,1)$ upto and inclusive of second degree terms. Hence compute $f(1.1, 0.9)$ approximately.

~~Ques 7~~ Ques 7 :→ If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$, Show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$$

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UNIT – II

Ques 8:→ If $u = x(1-r^2)^{-1/2}$, $v = y(1-r^2)^{-1/2}$, $w = z(1-r^2)^{-1/2}$ where $r^2 = x^2+y^2+z^2$, then show that

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = (1-r^2)^{-5/2}.$$

Ques 9:→ If $u = \sin^{-1}x + \sin^{-1}y$, $v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$ find $\frac{\partial(u,v)}{\partial(x,y)}$. Is u, v functionally related? If so find the relationship.

Ques 10:→ The angles of a triangle are calculated from the sides a, b, c. If small changes $\delta a, \delta b, \delta c$ are made in the sides, show that approximately

$$\delta A = \frac{a}{2\Delta}\left[\delta a - \delta b \cdot \cos C - \delta c \cdot \cos B\right]$$

where Δ is the area of the triangle and A, B, C are the angles opposite to a, b, c respectively. Verify that $\delta A + \delta B + \delta C = 0$

Ques 11:→ Test the function $f(x,y) = x^3 y^2 (6-x-y)$ for maxima and minima for points not at the origin.

Ques 12:→ A rectangular box, open at the top, is to have a given capacity. Find the dimensions of the box requiring least material for its construction.

Ques 13:→ The sum of three positive numbers is constant. Prove that their product is maximum when they are equal.

Ques 14:→ Find the maximum and minimum distances of the point (3, 4, 12) from the sphere

$$x^2+y^2+z^2=1.$$

Ques 15:→ Use Langrange's method of undetermined multipliers to find the minimum value of $x^2+y^2+z^2$ subject to conditions

$$x+y+z=1, \quad xyz+1=0$$

Ques 16:→ If $f(x,y) = x^2y/10$, compute the value of f when $x = 1.99$ and $y = 3.01$.

UNIT – III

Ques 17:→ State Cayley Hamilton theorm. Using this theorm find the inverse of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Ques 18:→ Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Also show that the matrix $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 - 8A^2 + 2A - I$ is equal to

$$\begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

Ques 19:→ Define a unitary matrix. If $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ is a matrix, then show that

$(I-N)(I+N)^{-1}$ is a unitary matrix, where I is an identity matrix.

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Ques20:- ~~Find~~ Verify the Cayley Hamilton theorm for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Hence Compute A^{-1}.

Ques21:- Find the eigen values and eigen vectors of the matrix.

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

Ques22:- Diagonalize the matrix $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Ques23:- Check the consistency of the following system of linear non-homogeneous equations and find the solⁿ, if exists

$$7x_1 + 2x_2 + 3x_3 = 16$$
$$2x_1 + 11x_2 + 5x_3 = 25$$
$$x_1 + 3x_2 + 4x_3 = 13$$

UNIT – IV

LONG ANSWER TYPE QUESTIONS

Ques 24:→ Evaluate $\iint_R (x+y)^2\,dx\,dy$ where R is the parallelogram in the xy plane with the vertices (1,0), (3,1), (2,2), (0,1). Using the transformation $u = x+y$, $v = x-2y$

Ques 25:→ Evaluate the following by changing into polar co-ordinates:

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2 \sqrt{x^2+y^2}\, dy\, dx$$

Ques 26:→ Evaluate $\iint (x+y)^2\,dx\,dy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Ques 27:→ Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate.

Ques 28:→ By changing the order of integration of $\int_0^\infty \int_0^\infty e^{-xy} \sin px\,dx\,dy$ show that

$$\int_0^\infty \frac{\sin px}{x}\,dx = \frac{\pi}{2}$$

Ques 29:→ Change into polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dy\,dx$

Hence show that $\int_0^\infty e^{-x^2}\,dx = \sqrt{\pi}/2$

Ques 30:- Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$ by changing to spherical polar coordinates.

Ques 31:→ State the Dirichlet's theorm for three variables. Hence evaluate the integral.

$$\iiint x^{l-1} y^{m-1} z^{n-1}\, dx\, dy\, dz$$, where x, y, z are all positive but limited conditions

$$\left(\frac{x}{a}\right)^{p} + \left(\frac{y}{b}\right)^{q} + \left(\frac{z}{c}\right)^{r} \le 1$$

UNIT-V

Ques 32:→ State Stoke's theorim and apply it to evaluate $\oint_C \vec{F}\cdot d\vec{r}$, where C is the boundary of rectangle $x = \pm a$, $y = 0$, $y = b$ and $\vec{F} = (x^2+y^2)\hat{i} - 2xy\hat{j}$

Ques 33:→ Establish the relation:

$$\text{Curl Curl } \vec{f} = \text{grad div } \vec{f} - \nabla^2 \vec{f}$$

Ques 34:→ Prove $\text{div}(\text{grad } r^n) = n(n+1) r^{n-2}$ where $r = \sqrt{x^2+y^2+z^2}$ Hence Show that

$$\nabla^2\left(\frac{1}{r}\right) = 0$$

Ques 35:→ State Gauss divergence theorm. Verify this theorm by evaluating the Surface integral as a triple integral

$$\iint_S (x^3\, dy\, dz + x^2 y\, dz\, dx + x^2 z\, dx\, dz)$$, where S is the closed surface consisting of the cylinder $x^2+y^2 = a^2$, $(0 \le z \le b)$