Longanswquestions (10 marks)

Q.1. If $u^3+v^3=x+y$, $u^2+v^2=x^3+y^3$, show that $\frac{\partial(u,v)}{\partial(x,y)}=\frac{y^2-x^2}{2uv(u-v)}$

Q.2. Find Taylorseries expansion of fn. $f(x,y)=e^{-x^2-y^2}\cos xy$ about point $x_0=0, y_0=0$ upto three terms.

Q.3. Find the minimum distance from the point $(1,2,0)$ to the cone $z^2=x^2+y^2$

Q.4. State Leibnitz theorem for n^{th} differential coeff. of the product of two fns.

If $y^{1/m}+y^{-1/m}=2x$, prove that

$$(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$$

Q.5. If $u=x\sin^{-1}(x/y)+y\sin^{-1}(y/x)$, find the value of

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$$

Q.6. If $x+y+z=u$, $y+z=uv$, $z=uvw$, then show that

$$\frac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v$$

Q.7. Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

Q.8. By changing the order of integration of

$$\int_0^\infty\int_0^\infty e^{-xy}\sin px\,dx\,dy,$$ show that

$$\int_0^\infty \frac{\sin px}{x}\,dx=\frac{\pi}{2}$$

Q.16. Define the eigen values, eigen vectors and the characteristics equation of a square matrix. Find the characteristic equation, eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 2 & 5 & 7 \\ 5 & 3 & 1 \\ 7 & 0 & 2 \end{bmatrix}$$

Q.17. Check the consistency of the following system of linear non-homogeneous equations and find the solution, if exists:

$$7x + 2y + 3z = 16$$
$$2x + 11y + 5z = 25$$
$$x + 3y + 4z = 13$$

Q.18. Find a matrix B which reduces A to the diagonal form by the transformation BAB^{-1} where

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

Hence find A^2.

Q.19. State Cayley Hamilton theorem. Verify it for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. Also find A^{-1}.

Q.20. Find the volume of largest parallelopiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$