Mathematics - I

Objective Questions Unit I

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Math - I

Q-1 What is the value of nth differential coefficient of $\frac{1}{1-5x+6x^2}$?

(a) $(-1)^n n! \left[\frac{2^{n}+1}{(2x-1)^{n+1}} - \frac{3^{n}+1}{(3x-1)^{n+1}} \right]$ (b) $(-1)^n ! \left[\frac{2^{n}-1}{(2x-1)} \right]$

(c) both (d) none of these

Q-2 If $y = A \sin nx + B \cos nx$, then $y_2 + n^2 y$ is equal to:

(a) 0 (b) 2 (c) 1 (d) 3

Q-3 If $y = a \sin(\log x)$; Then $x^2 y_2 + x y_1 + y$ is equal to;

(a) 2 (b) 0 (c) 3 (d) -1

Q-4 What is the value of $\frac{d^2y}{dx^2} + \frac{2a^2x^2}{y^5}$ when $y^3 - 3ax^2 + x^3 = 0$

(a) 0 (b) 1 (c) 2 (d) -2

Q-5 What is the value of $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{2}$ when $y = e^{-x} \sin x$

(a) 0 (b) 1 (c) 2 (d) -2

Q-6 What is the value of $\frac{d^2y}{dx^2}$ at the point $(\pi, 2)$ when $x = \theta - \sin\theta$, $y = 1 - \cos\theta$.

(a) $-\frac{1}{4}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) $-\frac{3}{2}$

Q-7 What is the nth differential coefficient of $x^{n-1} \log x$?

(a) $(n-1)/x$ (b) $(n-2)/x$ (c) $(n-3)/x$ (d) none of these.

Q-8 If $x = a(\cos\theta + \theta \sin\theta)$; $y = a(\sin\theta - \theta\cos\theta)$; then $\frac{d^2y}{dx^2}$ is

(a) $\frac{1}{a} \frac{\sec^3\theta}{\theta}$ (b) $a \frac{\sec^3\theta}{\theta}$ (c) $\theta \frac{\sec^3\theta}{a}$ (d) $a\theta \sec^3\theta$

Q-9 By Leibnitz's Theorem we find the nth differential coefficient of the ..... of two functions

(a) Sum (b) difference (c) Product (d) quotient.

Q-10 If $y = e^{\tan^{-1}x}$, then $(1+x^2)\, y_{n+2} + [2(n+1)x - 1]\, y_{n+1} = \ldots$

(a) $-n(n-1)y_n$ (b) $\frac{n}{2}(n+1)y_n$ (c) $-\frac{n}{2}(n-1)y_n$ (d) $-n(n+1)y_n$

Q-11 what is the differential value of $\frac{x^n}{1+\sin x}$?

(a) $x^{n-1}\{n(1+\sin x) - x\cos x\}/(1+\sin x)^2$

(b) $x^{n-1}\{n(1+\sin x)\}$ (c) $x^{n-1}\{n(1+\cos x - \sin x\}$

(d) none of these

Q-12 what is the differential coefficient of $\sqrt{\sin x}$?

(a) $\frac{1}{2}(\tan x)^{-1/2}\sec^2 x$ (b) $\frac{1}{3}(\tan x)^{-1/3}$ (c) $\frac{1}{2}(\tan x)^{-2/3}$ (d) none of these

Q-13 what is the differential coefficient of $\frac{1}{\log x}$?

(a) $-\frac{1}{x}(\log x)^{-2}$ (b) $-\frac{1}{2x}(\log x)$ (c) $\frac{1}{x}(\log x)$ (d) none of these

Q-14 what is differential value of $\frac{\sin x}{x}$ from first principle?

(a) $\frac{x\cos x - \sin x}{x^2}$ (b) $\frac{x\cos x + \sin x}{x}$ (c) $\frac{x\cos x - \sin x}{2x}$ (d) none of these

Q-15 what is the differential coefficient of $e^x \sin x$ at $x=0$ from first principles?

(a) 1 (b) 2 (c) 0 (d) -1

Q-16 Differentiate $x\log x$ from first principles?

(a) $\log x + 1$ (b) $\log x + 2$ (c) $\log x + 3$ (d) $\log x - 3$

Q-17 If $x = a\cos^3 t$ & $y = a\sin^3 t$ find $\frac{dy}{dx}$

(a) $-\tan t$ (b) $-\cot t$ (c) $-\cos t$ (d) none of these

Q-18 If $x = r\cos\theta$, $y = r\sin\theta$ find the value of $\left(\frac{\partial y}{\partial \theta}\right)_r$

(a) $r\cos\theta$ (b) $r\sin\theta$ (c) $-r\cos\theta$ (d) $-r\sin\theta$

Q-19 If $\theta = t^n e^{-r^2/4t}$ find what value of n will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$?

(a) $-\frac{3}{2}$ (b) $-\frac{5}{2}$ (c) $-\frac{5}{3}$ (d) none of these

Q-20 If $u = e^{m^2}(y-z)$, $y = m\sin x$ & $z = \cos x$ find $\frac{dy}{dx}$

(a) $e^{mx}(m^2+1)\sin x$ (b) $e^{m^2}(m^2+2)\sin x$ (c) $e^{mx}(m+2)$ (d) none of these

Q-21 If $z = \sqrt{x^2+y^2}$ and $x^3 + y^3 + 3axy = 5a^2$ find the value of $\frac{dz}{dx}$ when $x=a$, $y=a$

(a) 0 (b) 2 (c) 3 (d) 4

Q-22 Find $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$, when $\sin z = \frac{x^2 - y^2}{x^2 + y^2}$

(a) 0 (b) 2 (c) 3 (d) 4

Q-23 Change the independent variable from x to θ in the eqn $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ having given $x = \cos\theta$.

(a) $\frac{d^2y}{d\theta^2} + y = 0$ (b) $\frac{dy^2}{d\theta} = 0$ (c) $\frac{d^2y}{dx^2} + 1 = 0$ (d) none.

Q-24 Transformed equation of the equation $x^4\frac{d^2y}{dx^2} + 2x^3\frac{dy}{dx} + n^2y = 0$ by substitution $x = \frac{1}{z}$ is

(a) $\frac{d^2y}{dx^2} + ny = 0$ (b) $\frac{d^2y}{dz^2} + n^2y = 0$ (c) $\frac{d^2y}{dz^2} - ny = 0$ (d) $\frac{d^2y}{dz^2} - n^2y = 0$

Q-25 e^x Function expand by Maclaurin's Theorem is:

(a) $1 + x + \frac{x^2}{\lfloor 2} + \cdots \frac{x^n}{\lfloor n} + \cdots \infty$ (b) $1 + x + \frac{1}{x^2} \cdots \infty$

(c) $1 + \frac{1}{x} + x^2 + \cdots \infty$ (d) None of these

Q-26 Fill in the blanks of the following statements:

(i) If $y = \frac{ax+b}{cx+d}$, show that $2y_1y_3 = \cdots$

(ii) $\frac{(-1)^{n-1}(n-1)!\,a^n}{(ax+b)^n}$ is the nth differential coefficient of ....

(iii) If $y = (ax+b)^{-1}$, Then $D^n(ax+b)^{-1} = \cdots$

(iv) The n^{th} differential coefficient of $e^x \sin^2 x = \cdots$

(v) If $y = a\cos(\log x) + b\sin\log x$; then $x^2y_2 + xy_1 = \cdots$

(vi) If $u = e^{my}\cos mx$ Then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \cdots$

(vii) If $u = \tan^{-1}\frac{x^2+y^2}{x+y}$ Then $\frac{\partial u}{\partial x} = \cdots$

(viii) An expression in which every term is of the same degree is called a .... function.

(ix) If $u = f(x,y)$ where $x = \phi(t)$ & $y = \psi(t)$, Then $\frac{du}{dt} = \cdots$

(x) If $u(x,y)$ is a homogeneous function of x & y of degree n, Then $x\frac{\partial}{\partial x}(u_x) + y\frac{\partial}{\partial y}(u_x) = \cdots$ where $u_x = \frac{\partial u}{\partial x}$

Unit II

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Math - I

Q-1 If $x = r\cos\theta$, $y = r\sin\theta$ find $\frac{\partial(x,y)}{\partial(r,\theta)} = \ldots$

(a) r (b) $r/2$ (c) $r/3$ (d) none of these

Q-2 If $x = u(1+v)$ & $y = v(1+u)$, find the Jacobian of x, y w.r.t. u, v.

(a) $1+u+v$ (b) $1-u-v$ (c) $1-u+v$ (d) none of these.

Q-3 If $u = \frac{y^2}{2x}$ $v = \frac{x^2+y^2}{2x}$ Then

(a) $\frac{\partial(u,v)}{\partial(x,y)} = \frac{y}{2x}$ (b) $\frac{\partial(u,v)}{\partial(x,y)} = \frac{x^3}{2y^3}$ (c) $\frac{\partial(u,v)}{\partial(x,y)} = \frac{2x}{y}$

(d) none of these.

Q-4 If u & v are the function of x & y Then $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = \ldots$

(a) 1 (b) 0 (c) 2 (d) none of these

Q-5 If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ Then $\frac{\partial l}{\partial(r,\theta,\phi)} = \ldots$

(a) $r^2\sin\theta$ (b) $r\sin\theta$ (c) r (d) none of these.

Q-6 ~~Cho~~ what is the maximum condition in terms of p, q, r, s & t ?

(a) $p=0$ $q=0$ $rt-s^2>0$ $r<0$

(b) $p=0$ $q=0$ $rt-s^2>0$ $r>0$

(c) $p=0$ $q=0$ $rt-s^2<0$ $r<0$

(d) $p=0$ $q=0$ $rt-s^2<0$ $r>0$

Q-7 The three sides of a Trapezium are equal each being 6 cm long. Find the maximum area of the trapezium.

(a) $3\sqrt{3}$ cm^2 (b) $\sqrt{3}$ cm^2 (c) $\sqrt{2}$ cm^2 (d) 2 cm^2

Q-8 Find the area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(a) 2 ab (b) 3 ab (c) 10 ab (d) 11 ab

Q-9 choose the true & False in the following statements:

(i) If $y_1 = \frac{x_2x_3}{x_1}$, $y_2 = \frac{x_1x_3}{x_2}$, $y_3 = \frac{x_1x_2}{x_3}$ Then $J(y_1 y_2 y_3) = 4$

(ii) If $x+y+z = u$, $y+z = uv$, $z = uvw$ The value of the Jacobian of x, y, z is u^2v with regard to u, v, w.

(iii) If $x = r\cos\theta$ & $y = r\sin\theta$, $r = au$ & $\theta = bv$ Then $\frac{\partial(x,y)}{\partial(u,v)}$ is abr.

(iv) If $u = x+3y+2z$; $v = 3x+4y-2z$ $w = 11x+18y-2z$ Then $2u+3v = w$ is the functional relation.

(v) If $u = x+y+z$, $v = xy+yz+zx$ & $w = x^3+y^3+z^3-3xyz$ Then a relation between them is $w+3uv$.

(vi) The functions $u = 3x+3y-2$, $v = x-2y+2$, $w = x[x+2y-2]$ are not independent.

(vii) The maximum & minimum values of u in the $u = x^2y^2-5x^2-8xy-5y^2$ is $x=y=0$ gives a maximum.

(viii) The maxima & minimma of $u = \sin x \sin y \sin z$ where $x+y+z = \pi$ is maxima at $x=y=z=\pi/3$

(ix) Given $x+y+z = a$, $a^3/27$ is maximum value of xyz.

(X) If u_1, u_2 & u_3 are functions of three independent variables x_1, x_2, x_3 connected by the eqns. $F_1(u_1,u_2,u_3,x_1,x_2,x_3) = 0$ $F_2(u_1,u_2,u_3,x_1,x_2,x_3) = 0$ & $F_3(u_1,u_2,u_3,x_1,x_2,x_3) = 0$ Then

$$\frac{\partial(u_1,u_2,u_3)}{\partial(x_1,x_2,x_3)} = \frac{\partial(F_1,F_2,F_3)/\partial(x_1,x_2,x_3)}{\partial(F_1,F_2,F_3)/\partial(u_1,u_2,u_3)}$$

Unit - III

Q-1 If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then A is

(a) diagonal matrix (b) scalar matrix (c) unit matrix (d) none of these

Q-2 If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 & -6 \\ 0 & -1 & 3 \end{bmatrix}$ Then $3A - 4B$ is equal to :

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 & 27 \\ 0 & 1 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 5 & -5 \\ 0 & -2 & 8 \end{bmatrix}$ (d) none of these.

Q-3 If $\begin{bmatrix} 5 & k+2 \\ k+1 & -2 \end{bmatrix} = \begin{bmatrix} k+3 & -4 \\ 3 & -k \end{bmatrix}$, Then k =

(a) 0 (b) 2 (c) -2 (d) 1

Q 4 If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ Then

(a) AB, BA exist & are equal (c) AB exists & BA doest not exist

(b) AB ; BA exist & are not equal (d) AB does not exist & BA exist

Q-5 If A is 3×4 matrix & B is a matrix such that $A'B$ & BA' are both defined. Then B is of the type ;

(a) 3×4 (b) 3×3 (c) 4×4 (d) 4×3

Q-6 If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ Then $A^2 =$ ...

(a) A (b) $-A$ (c) 2A (d) $-2A$

Q-7 If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ Then $A^2 =$

(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Q-8 The matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is known as:

(a) diagonal matrix (b) upper triangular matrix

(c) Symmetric matrix (d) Skew - symmetric matrix.

Q-9 If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, Then A^2 is equal to :

(a) A (b) $-A$ (c) null matrix (d) I

Q-10 If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, Then AB =

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Q-11 If A is singular matrix, then adj A is

(a) Singular (b) non singular (c) Symmetric (d) not defined.

Q-12 The order of $[x\ y\ z]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is:

(a) 3×3 (b) 1×1 (c) 1×3 (d) 3×1

Q-13 If A is any non-zero square matrix of order n, $A(\text{adj } A)$ is equal to:

(a) I (b) $|A|\,I$ (c) $|A|^n\,I$ (d) none of these

Q-14 The matrix $\begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is invertible if:

(a) $\lambda \neq -15$ (b) $\lambda \neq -17$ (c) $\lambda \neq -16$ (d) $\lambda \neq -18$

Q-15 The inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$ is:

(a) $\begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ac-b & -c & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ -a & 0 & 0 \\ b & -c & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ -a & 0 & 0 \\ ac & a & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -a & ac-b \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}$

Q-16 If the system of the equations $x+2y-3z=1$, $(p+2)z=3$, $(2p+1)y+z=2$ is inconsistent, then the value of p is:

(a) -2 (b) $-\frac{1}{2}$ (c) 0 (d) 2

Q-17 If the system of equations $ax+y=1$, $x+2y=3$, $2x+3y=5$ are consistent, a is given by:

(a) 0 (b) 1 (c) 2 (d) none of these.

Q-18 A & B are two non-zero square matrices such that $AB=0$. Then

(a) both A & B are singular.

(b) either of them is singular

(c) neither matrix is singular

(d) none of these

Q-19 From the matrix equation $AB=BC$ we can conclude $B=C$ provided:

(a) A is singular (B) A is non-singular (c) A is symmetric (d) none

Q-20 The system of linear equations $x+y+z=2$, $2x+y-z=3$, $3x+2y+kz=4$ has a unique solution of

(a) $k\neq0$ (b) $-1<k<1$ (c) $-2<k<2$ (d) $k=0$