Unit IV

Q-1 Find the following statements are True or False

(a) If x_1, x_2 are the function of y and y_1, y_2 are constant Then

$$\iint_R f(x,y)\, dx\, dy = \int_{y_1}^{y_2} \int_{x_1=\phi_1(y)}^{x_2=\phi_2(y)} f(x,y)\, dy\, dx$$

(b) $$\int_0^1 \int_0^x e^{y/x}\, dy\, dx = \frac{1}{2}(e-1)$$

(c) $$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy\, dx}{1+x^2+y^2} = \pi \log(\sqrt{2}+1)$$

(d) $$\int_0^a \int_0^{\sqrt{a^2-y^2}} dx\, dy = \frac{\pi a^2}{4}$$

Ans: (a) F (b) T (c) F (d) T

Q-2 Fill in the blanks

(a) $$\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x\, dy\, dx}{\sqrt{x^2+y^2}} = \text{-----}$$

(b) $$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy = \text{----}$$

(c) $$\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx = \text{----}$$

(d) $$\int_0^2 \int_1^{e^x} dy\, dx = \text{----}$$

Ans. (a) $\frac{4}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{3}{8}$ (d) (e^2-3)

Q-3 Area bounded by parabola $y^2 = 4ax$ & its latus rectum is

(a) $\frac{8a}{3}$ (b) $\frac{8a^2}{3}$ (c) $\frac{4a}{3}$ (d) $\frac{4a^2}{3}$

Q-4 $$\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dx\, dy\, dz = \text{---}$$

(a) 48 (b) 49 (c) 50 (d) 51

Q-5 $$\int_0^1 dx \int_0^2 dy \int_1^2 x^2 yz\, dz = \text{....}$$

(a) 0 (b) 1 (c) 2 (d) 3

Q-6 $$\int_0^2 \int_0^1 \int_{-1}^1 (x^2+y^2+z^2)\, dx\, dy\, dz = \text{....}$$

(a) 4 (b) 5 (c) 6 (d) 7

Ans Q-3 (b) Q-4 (a) Q5 (b) Q-6 (c)

Q-6 Fill in the blanks of the following Statements;

(a) $\iiint dx\,dy\,dz$ is called - - - - -

(b) Volume of the region bounded by the surface $y=x^2, x=y^2$ and the planes $z=0$, $z=3$ is - - - . .

(c) Surface area of the cylinder $x^2+z^2=4$ inside the cylinder $x^2+y^2=4$ is - - -

(d) Γn = - - - -

(e) $\int_0^\infty \sqrt[4]{x}\, e^{-\sqrt{x}}\, dx$ = - - -

Ans (a) Volume (b) 1 (c) 32 (d) $\int_0^\infty e^{-x} x^{n-1} dx$

(e) $\frac{3}{2}\sqrt{\pi}$

Q-7 Find the statements are True or False

(a) $\Gamma\left(-\frac{1}{2}\right) = -2\sqrt{\pi}$

(b) $1.3.5.\ldots(2n-1) = \dfrac{2^n\,\Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}}$

(c) $B(l,m) = B(m,l)$

(d) $\int_0^\infty e^{-y^{1/m}} dy = m\,\Gamma m$

(f) $\Gamma m\,\Gamma\left(m+\frac{1}{2}\right) = \dfrac{\sqrt{\pi}}{2^{2m-1}}\,\Gamma 2m$, $m>0$.

(g) $\Gamma n\,\Gamma(1-n) = \dfrac{\pi}{\sin n\pi}$

(h) $\iiint_V x^{l-1} y^{m-1} z^{n-1}\, dx\,dy\,dz = \dfrac{\Gamma l\,\Gamma m\,\Gamma n}{\Gamma(l+m+n+1)}$ where V is the region $x\geq 0$, $y\geq 0$, $z\geq 0$, $x+y+z\leq 1$ & l, m, n are all positive.

Ans. (a) T (b) T (c) T (d) T (e) T (f) T

(g) T (h) T

Q 8 For $m>0$ $n>0$

(a) $B(m,n) = \frac{\Gamma m}{\Gamma n}$ (b) $\frac{\Gamma m \, \Gamma n}{\Gamma m+n}$ (c) $B(m,n) = \frac{\Gamma n}{\Gamma m}$ (d) $B(m,n) = \frac{\Gamma m+n}{\Gamma m \Gamma n}$

Q-9 The value of the integral $\int_0^\infty e^{-x} x^{-1/2} dx$ is

(a) $\sqrt{\pi}/2$ (b) $\pi/2$ (c) $\sqrt{\pi}$ (d) π

Q-10 For $m>0$ $n>0$

(a) $B(m,n) = \int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}} dx$ (b) $B(m,n) = \int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}} dx$

(c) $B(m,n) = \int_0^1 \frac{x^{n-1}}{(1+x)^{m+n}} dx$ (d) $B(m,n) = \int_0^\infty \frac{x^m}{(1+x)^{m+n}} dx$

Q-11 For $a>0$ $n>0$

(a) $\int_0^\infty e^{-ax} x^{n-1} dx = \frac{\Gamma n}{a^n}$ (b) $\int_0^1 e^{-ax} x^{n-1} dx = \frac{\Gamma n}{a^n}$

(c) $\int_0^\infty e^{-ax} x^{n-1} dx = \frac{\Gamma n}{a}$ (d) none of these.

Q-12 If $0<n<1$ Then

(a) $\Gamma n \, \Gamma 1-n = \frac{\pi}{\sin n\pi}$ (b) $\Gamma n \, \Gamma 1-n = \frac{\pi}{\sin \pi}$

(c) $\Gamma 1+n \, \Gamma 1-n = \frac{\pi}{\sin n\pi}$ (d) none of these

Ans: 8. (b) 9. (c) 10. (a) 11. (a) 12. (a)

Q-13 Fill in the blanks of the following statements;

(i) ~~The~~ The definite Integral $\int_0^1 x^{m-1}(1-x)^{n-1} dx$ for $m>n$ is called the ......

(ii) The definite integral $\int_0^\infty e^{-x} x^{n-1} dx$, for $n>0$ is called the ......

(iii) $B(m,n) = \frac{\Gamma m \, \Gamma n}{\ldots\ldots}$

(iv) $\frac{B(m+1,n)}{B(m,n)} = \frac{m}{\ldots\ldots}$

(v) For $m>0$ $n>0$, $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx = \ldots\ldots$

(vi) For $m>0$ $n>0$ $\int_0^\infty \frac{x^{m-1} - x^{n-1}}{(1+x)^{m+n}} dx = \ldots\ldots$

(vii) For $n>0$, $\Gamma n+1 = \ldots \Gamma n$

(viii) $\Gamma \frac{1}{2} = \ldots\ldots$

(ix) If n is +ve integer then $\Gamma n = \text{---}!$

(x) If $m > -1$, $n > -1$ Then $\int_0^{\pi/2} \cos^m\theta \sin^n\theta \, d\theta = \dfrac{\Gamma\frac{m+1}{2} \cdot \Gamma\frac{n+1}{2}}{\text{------}}$

(xi) $\int_0^{\infty} e^{-x^2} \, dx = \text{-----}$

(xii) For $m>0$ $\quad \Gamma m \, \Gamma\left(m+\frac{1}{2}\right) = \dfrac{\sqrt{\pi}}{\text{---}} \, \Gamma 2m$.

(xiii) The value of $\Gamma\frac{1}{3} \, \Gamma\frac{2}{3}$ is ----

Ans: (i) Beta function (ii) Gamma function (iii) $\Gamma m+n$ (iv) $m+n$ (v) $B(m,n)$ (vi) 0 (vii) n (vii) $\sqrt{\pi}$ (ix) $(n-1)$ (x) $2\,\Gamma\frac{m+n+2}{2}$ (xi) $\frac{\sqrt{\pi}}{2}$ (xii) 2^{2m-1} (xiii) $\frac{2\pi}{\sqrt{3}}$.

Unit-V

Q-1 Fill in the blanks of the following statements:

(a) $\frac{d}{dt}(\vec{F} \times \vec{G}) = \ldots\ldots$

(b) $i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} = \ldots\ldots$

(c) If r is the magnitude of position vector ~~then~~ $r = xi + yj + zk$ Then $\nabla^2\left(\frac{x}{r^3}\right) = \ldots$

(d) $\nabla \times [f(r)\,\vec{r}] = \ldots\ldots$

(e) $\vec{\nabla} \cdot \left[\frac{f(r)}{r}\vec{r}\right] = \ldots\ldots$

(f) directional derivative of $\phi = \ldots\ldots$

Q-2 If $u = x + y + z$, then ∇u is

(a) $xi + yj + zk$ (b) $\frac{xi + yj + zk}{\sqrt{x^2+y^2+z^2}}$ (c) $\frac{i+j+k}{\sqrt{3}}$ (d) $i + j + k$

Q-3 If $\phi = 3x^2y - y^3z^2$, grad ϕ at the point $(1,-2,-1)$ is

(a) $9i + 5j + 7k$ (b) $12i + 9j + 16k$ (c) $-12i + 9j - 16k$ (d) $-12i - 9j - 16k$

Q-4 A unit vector normal to the surface $x^2 + 3y^2 + 2z^2 = 6$ at $P(2,0,1)$ is

(a) $i + j + k$ (b) $\frac{i+j+k}{\sqrt{3}}$ (c) $\frac{i+k}{\sqrt{2}}$ (d) $i + k$

Q-5 If $\vec{r} = xi + yj + zk$ Then the direction derivative of $\frac{1}{r}$ is

(a) $-\frac{xi + yj + zk}{(x^2+y^2+z^2)^{3/2}}$ (b) $\frac{xi+yj+zk}{\sqrt{x^2+y^2+z^2}}$ (c) $-\frac{xi+yj+zk}{(x^2+y^2+z^2)^2}$ (d) $-\left[\frac{xi+yj+zk}{x^2+y^2+z^2}\right]^2$

Q-6 At any point of the curve $x = 3\cos t$ $y = 3\sin t$, $z = 4t$ then tangent vector is

(a) $3\cos t\, i + 3\sin t\, j + 4t\, k$ (b) $3\sin t\, i + 3\cos t\, j + 4k$ (c) $-3\sin t\, i + 3\cos t\, j + 4k$
(d) $-3\sin t\, i - 3\cos t\, j - 4k$

Q-7 $\phi = x^2 + y^2 + z^2$ at the point $(2,2,1)$ in the direction of $2i + 2j + k$ the directional derivative is

(a) 2 (b) 6 (c) 8 (d) 1

Q-8 when directional derivative of $f(x,y) = \frac{x^2-y^2}{xy}$ at $(1,1)$ zero direction is

(a) $i+j+k$ (b) $\frac{i+k}{\sqrt{2}}$ (c) $\frac{i+j}{\sqrt{2}}$ (d) $i+j$

Q-9 If $\phi(x,y,z) = 3xz^2y - y^3z^2$ Then grad ϕ at the point $(1,-2,-$ is

(a) $6xz\,i + 3yz\,j + 2zx\,k$ (b) $-16i - 9j - 4k$

(c) $16i - 9j + 4k$ (d) $16i + 9j + 4k$

Q-10 A unit normal vector to the surface $z^2 = x^2 + y^2$ at the point $(1,0,-1)$ is

(a) $i+j$ (b) $\frac{i+j}{\sqrt{2}}$ (c) $i+k$ (d) $\frac{i+k}{\sqrt{2}}$

Q-11 If fluid is compressible ... and volume is V Then

(a) grad $V = 0$ (b) Curl $V = 0$ (c) div $\vec{V} = 0$ (d) none of the

Q-12 If $\vec{v} = \frac{xi+yj+zk}{\sqrt{x^2+y^2+z^2}}$, find the value of div $\vec{v}$.

(a) $\frac{1}{\sqrt{x^2+y^2+z^2}}$ (b) $\frac{2}{\sqrt{x^2+y^2+z^2}}$ (c) $\frac{xi+yj+zk}{\sqrt{x^2+y^2+z^2}}$ (d) 0

Q-13 If $u = x^2+y^2+z^2$ & $\vec{r} = xi+yj+zk$, Then div $(u\vec{r})$ is

(a) u (b) $5u$ (c) $6u$ (d) $2u$

Q-14 A vector $\vec{V}$ is solenoidal if

(a) curl $\vec{V} = 0$ (b) div $\vec{V} = 0$ (c) div $V \neq 0$ (d) grad $\vec{V} = 0$

Q-15 A field $f(x,y,z)$ describing a motion is irrotational if

(a) $\nabla \times \nabla f = 0$ (b) $\nabla \times \nabla \times f = 0$ (c) $(\nabla \times \nabla) f \neq 0$ (d) none of these

Q-16 If $\vec{F}$ is conservative vector field and ϕ is called the scalar potential Then $\vec{\nabla} \times \vec{F} = \vec{\nabla} \times \vec{\nabla}\phi = \ldots$

(a) 0 (b) 1 (c) 2 (d) none of these

Q-17 A force $yi + xj$ which displaces a particle from origin to a point $(i+j)$ Then work done is

(a) 0 (b) 1 (c) 2 (d) 3

Q-18 Evaluate $\iint_S (yz\hat{i} + zx\hat{j} + xy\hat{k}) \cdot d\vec{S}$. where S is the surface of the sphere $x^2+y^2+z^2=a^2$ in the first octant.

(a) $\frac{3a^4}{8}$ (b) 0 (c) both (a) & (b) (d) none of these

Q-19 Evaluate $\int_c (x^2 y\,dx + x^2 dy)$ where c is the boundary described counter clockwise of the triangle with vertices (0,0), (1,0) & (1,1).

(a) 0 (b) $\frac{4}{3}$ (c) $-\frac{1}{2}$ (d) $\frac{5}{12}$

Q-20 If Gauss Theorem of divergence relation between is

(a) Line & surface Integral

(b) Line & plane Integral

(c) Surface & Volume Integral

(d) Line & Volume Integral.