UNIT-I SHORT ANSWER TYPE QUESTIONS

Ques1:→ Find the n^{th} derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$.

Ques2:→ If $y = x \log \frac{x-1}{x+1}$, show that

$$y_n = (-1)^{n-2}(n-2)!\left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$$

Ques3:→ If $y = (x^2-1)^n$ Prove that

$$(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$$

Hence if $P_n = \frac{d^n}{dx^n}(x^2-1)^n$, show that

$$\frac{d}{dx}\left\{(1-x^2)\frac{dP_n}{dx}\right\} + n(n+1)P_n = 0$$

Ques4:→ If $\theta = t^n e^{-\frac{r^2}{4t}}$, find the value of n which will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{\partial\theta}{\partial t}$

Ques5:→ State Euler's theorm of differential Calculus. Hence verify the theorm for the function $u = \log \frac{x^2+y^2}{xy}$.

Ques6:→ If $u = f(r)$ and $x = r\cos\theta$, $y = r\sin\theta$. then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$

Ques7:→ If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ then show that

$$x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$$

Ques8:→ If $y = a\cos(\log x) + b\sin(\log x)$, Show that

$$x^2 y_{n+2} + (2n+1)x y_{n+1} + (n^2+1)y_n = 0$$

Ques 9: If $u = \tan^{-1} \frac{x^3+y^3}{x-y}$ then prove that

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u$
$= 2\cos 3u \sin u$

Ques 10: If $u = \sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} + 3\tan u = 0$$

Ques 11: Expand $\frac{(x+h)(y+k)}{x+h+y+k}$ in powers of h, k upto and inclusive of the second degree terms.

Ques 12: Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y-2)$ using Taylor's theorem.

Ques 13: Expand $e^x \cos y$ about the point $(1, \pi/4)$.

UNIT – II

Ques 14: If $u = xyz$, $v = x^2+y^2+z^2$, $w = x+y+z$. Find the Jacobian $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

(ii) If $x+y+z = u$, $y+z = uv$, $z = uvw$ then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$.

Ques 15: If $u = \frac{x+y}{z}$, $v = \frac{y+z}{x}$, $w = \frac{y(x+y+z)}{xz}$, then show that u, v, w are not independent and find the relation between them.

Ques 16:→ A balloon is in the form of right circular cylinder of radius 1.5 m and length 4 m and is surmounted by hemispherical ends. If the radius is increased by 0.01 m and length by 0.05 m, find the percentage change in the volume of balloon.

Ques 17:→ In estimating the cost of a pile of bricks measured as 6 m × 50 m × 4 m, the tape is stretched 1% beyond the standard length. If the count is 12 bricks in 1 m^3 and bricks cost Rs 100 per 1,000. find the approximate error in cost.

Ques 18:→ In determining the specific gravity by the formula $S = \frac{A}{A-w}$ where A is the weight in air and w is the weight in water; A can be read within 0.01 gm and w within 0.02 gm. Find approximately the maximum error in S if the readings are A = 1.1 gm, w = 0.06 gm. Find also the maximum relative error.

Ques 19:→ Find the extreme values of function x^3+y^3-3axy.

Ques 20:→ Test the function $f(x,y) = (x^2+y^2)e^{-(x^2+y^2)}$ for maxima and minima for points not on the circle $x^2+y^2=1$

Ques 21:→ A rectangular box, open at the top, is to have a volume of 32 c.c. Find the dimensions of the box requiring least material for its

Ques22:→ Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Ques23:→ The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature at the surface of a unit sphere $x^2+y^2+z^2=1$

Ques24:→ A tent of given volume has a square base of side 2a and has its four sides of height b vertical and is surmounted by a pyramid of height h. Find the values of a and b in terms of h so that the canvas required for its construction be minimum.

Ques25:→ Find the minimum distance from the point (1, 2, 0) to the cone $z^2 = x^2+y^2$.

Ques26:→ $y = \cos(m \sin^{-1} x)$, prove that
$$(1-x^2)\,y_{n+2} - x\,y_{n+1} + (m^2-n^2)\,y_n = 0$$

~~Ques27: Show that~~

UNIT – III SHORT ANSWER TYPE QUESTIONS

Ques 27:→ Show that the vectors $x_1 = (1, 2, 4)$, $x_2 = (2, -1, 3)$, $x_3 = (0, 1, 2)$, $x_4 = (-3, 7, 2)$ are linearly dependent find the relation between them.

Ques 28:→ Define Unitary matrix. How that matrix $\begin{bmatrix} \alpha+i\gamma & -\beta+i\delta \\ \beta+i\delta & \alpha-i\gamma \end{bmatrix}$ is a unitary matrix If $\alpha^2+\beta^2+\gamma^2+\delta^2 = 1$

Ques 29:→ Show that the sum of the eigen values of a matrix is equal to the sum of the elements of its principle diagonal and Show that for any square matrix A, the product of all eigen values of A is equal to det (A).

Ques 30:→ Test for Consistency the following system of equations and If Consistent solve them :-

$$x_1 + 2x_2 - x_3 = 3$$
$$-x_2 + 2x_3 = 1$$
$$+2x_1 - 2x_2 + 3x_3 = 2$$
$$x_1 - x_2 + x_3 = -1$$

Ques 31:→ Find the value of λ for which the vectors $(1, -2, \lambda)$, $(2, -1, 5)$ and $(3, -5, 7\lambda)$ are linearly dependent.

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UNIT – IV

Ques 32:→ Let D be the region in the first quadrant bounded by the curves $xy = 16$, $x = y$, $y = 0$ & $x = 8$. Sketch the region of integration of the following integral $\iint_D x^2\,dx\,dy$ and evaluate it by expressing it as an appropriate repeated integral.

Ques 33:→ Let D be the region in the first quadrant bounded by $x = 0$, $y = 0$ and $x + y = 1$. Change the variables x, y to (u, v) where $x + y = u$, $y = uv$ and evaluate

$$\iint_D xy\,(1-x-y)^{1/2}\,dx\,dy$$

Ques 34:→ Using the transformation $x + y = u$, $y = uv$ show that

$$\int_0^1 \int_0^{1-x} e^{y/(x+y)}\,dy\,dx = \frac{1}{2}(e-1)$$

UNIT – V

Ques 35:→ Find the directional derivative of $(1/r)$ in the direction $\vec{r}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

Ques 36:→ Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at point $(2, -1, 2)$.

Ques 37:→ Use divergence theorm to show that,

$$\iint_S \nabla (x^2+y^2+z^2)\, d\vec{s} = 6V$$ where S is any closed surface enclosed by volume V.

Ques 38:→ Using Green's theorm evaluate ~~$\int_C x^2y\,dx$~~ $\int_C x^2y\,dx + x^2dx + x^2dy$ where C is the boundary described clockwise of the triangle with vertices (0,0), (1,0), (1,1).

Ques 39:→ Find the direction derivative of $\phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point P(1,1,1) in the direction of the line

$$\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}.$$

Ques 40:→ Find $\iint \bar{F}\cdot\hat{n}\,ds$, where

$$\vec{F} = (2x+3z)\hat{i} - (xz+y)\hat{j} + (y^2+2z)\hat{k}$$

and S is the surface of the sphere having centre (3,−1,2) and radius 3.

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