- Short-answers type questions: (5 marks)

Q.1. Find rank of a matrix

$$\begin{bmatrix} 2 & 3 & -2 & 4 \\ 3 & -2 & 1 & 2 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{bmatrix}$$

Q.2 Find the value of λ for which the vectors $(1,2,\lambda)$; $(2,-1,5)$ and $(3,-5,7\lambda)$ are linearly dependent.

Q.3 Diagonalize the matrix $\begin{bmatrix} 1 & 6 & 3 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Q.4. Find the rank of the matrix by reducing it to normal form:

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 3 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}$$

Q.5. Find for what values of λ and μ, the system of linear equation:

$$x+y+z=6$$
$$x+2y+5z=10$$
$$2x+3y+\lambda z=\mu$$

has (i) a unique solution (ii) no solution (iii) infinite solution.

Q.6. Define Hermitian and skew Hermitian matrices.
If $H = \begin{bmatrix} 3 & 5+2i & -3 \\ 5-2i & 7 & 4i \\ -3 & -4i & 5 \end{bmatrix}$, show that H is a Hermitian matrix. Verify that iH is a skew-Hermitian matrix.

Q.7. If $y = \sin\left[\log(x^2+2x+1)\right]$, then prove the following:

(i) $(1+x)^2 y_2 + (1+x) y_1 + 4y = 0$

(ii) $(1+x)^2 y_{n+2} + (2n+1)(1+x) y_{n+1} + (n^2+4) y_n = 0$

Q.8. If $\theta = t^n e^{-r^2/4t}$, find the value of n which will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$

Q.9. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

Q.10. Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ using Taylor's theorem.

Q.11. If $u = \sin^{-1}\left[\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right]$

show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} + 3\tan u = 0$

Q.12. Expand x^y in powers of $(x-1)$ and $(y-1)$ upto the third degree terms.

Q.13. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that

(i) div. $\vec{r} = 3$ (ii) Curl $\vec{r} = 0$

Q.14. Evaluate the following by changing into polar co-ordinates:

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2\sqrt{x^2+y^2}\, dy\, dx$$

Q.15. Find the area enclosed between the parabola $y = 4x - x^2$ and the ~~strg~~ line $y = x$.

Q.16. Find the directional derivative of $F(x,y,z) = x^2yz + 4xz^2$ at $(1,-2,1)$ in the direction $2\hat{i} - \hat{j} - 2\hat{k}$. Find the greatest rate of increase of F.

Q.17. A fluid motion is given by $\vec{V} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ Is this motion irrotational? If so, find the velocity potential.

Q.18. The period of simple pendulum with small oscillations is given by $T = 2\pi\sqrt{l/g}$. Find the maximum percentage error in T due to possible error of 1% in l and 2% in g.

Q.19. Trace the curve $y^2(a-x) = x^3$, $a>0$

Q.20. If $u(x,y,z) = \log(\tan x + \tan y + \tan z)$, prove that
$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$$

Q.21. Verify Stoke's theorem for $F = (x^2+y^2)i - 2xy\,j$ taken around the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$.

Q.22. Trace the curve $y^2(a+x) = (a-x)x^2$

Q.23. Evaluate $\displaystyle\int_0^2\int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$, by changing it to polar co-ordinates.

Q.24. Determine the constant 'a' so that the vector $\vec{F} = (x+3y)i + (y-2z)j + (x+az)k$ is solenoidal.